Cryptanalysis of two chaotic encryption schemes based on circular bit shift and XOR operations

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Abstract

Recently two encryption schemes were proposed by combining circular bit shift and XOR operations, under the control of a pseudorandom bit sequence (PRBS) generated from a chaotic system. This paper studies the security of these two encryption schemes and reports the following findings: 1) there exist some security defects in both schemes; 2) the underlying chaotic PRBS can be reconstructed as an equivalent key by using only two chosen plaintexts; 3) most elements in the underlying chaotic PRBS can be obtained by a differential known-plaintext attack using only two known plaintexts. Experimental results are given to demonstrate the feasibility of the proposed attack.

Key words: chaos, encryption, delayed chaotic neural network, cryptanalysis, chosen-plaintext attack, known-plaintext attack, differential cryptanalysis
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1 Introduction

In the past three decades, many digital chaotic ciphers [1–9] and analog chaos-based secure communication schemes [2, 3, 10–12] have been proposed, trying to explore the intrinsic relationship between chaos and cryptography. However, due to the lack of a strict scrutiny on the security, most chaos-based cryptosystems have been found insecure against various attacks [6–9, 12, 13].

In [14], a new block encryption scheme was proposed by combining circular bit shift and XOR operations, under the control of a pseudorandom bit sequence (PRBS) generated from the chaotic logistic map. Later, in [15], the above encryption scheme was further modified [1], by adopting some alterations such as replacing the logistic map with a delayed chaotic neural network (DCNN).

In [14, Sec. 6.2], it is pointed out that the encryption scheme based on the logistic map is not secure enough against chosen-plaintext attack, because there exists some information leakage about the chaotic trajectory involved. Then, the authors of [14] suggested using key switching and/or "cycling chaos" [16] as remedies to further improve the security. However, as we show below in this paper, the information leakage is actually not the main reason why the encryption scheme is not secure against chosen-plaintext attack. We further show that both encryption schemes are not only insecure against chosen-plaintext attack, but also insecure against a differential known-plaintext attack. In addition, we will point out some other security defects existing in the design of these two chaos-based encryption schemes.

The rest of the paper is organized as follows. The next section gives a brief introduction to the two encryption schemes. Some security problems existing in both of the two encryption schemes are reported in Sec. 3. The main cryptanalytic results about plaintext attacks are given in Sec. 4, with some experimental results for demonstration. The last section concludes this paper.

2 Two Chaotic Encryption Schemes

To facilitate the following description of the two encryption schemes, the definitions of circular bit shift operations and some notations are first introduced.

Definition 1 *Assuming that* $L \in \mathbb{Z}^+$, $x \in \mathbb{Z}$ *and* $a = \sum_{i=0}^{L-1} (a_i \cdot 2^i) \in$

[1] Note that the authors of [15] did not make clear that their work is a modification of the one proposed in [14]. However, it is obvious that the DCNN-based scheme in [15] was originated from the work reported in [14] because the encryption procedures of the two schemes are exactly the same except for some minor modifications.

$\{0, \cdots, 2^L - 1\}$, *where* $a_i \in \{0, 1\}$, *the* L*-bit left and right circular bit shift operations are defined as follows:* $a \lll_L x = a \ggg_L (-x) = \sum_{i=0}^{L-1} \left(a_i \cdot 2^{(i+x) \bmod L} \right)$ *and* $a \ggg_L x = a \lll_L (-x) = \sum_{i=0}^{L-1} \left(a_i \cdot 2^{(i-x) \bmod L} \right)$.

From Definition 1, one can easily verify some simple properties about the circular bit shift operations: 1) $\forall\ x \equiv 0 \pmod{L}$, $a \lll_L x = a \ggg_L x = a$; 2) $\forall x_1 \equiv x_2 \pmod{L}$, $a \lll_L x_1 = a \lll_L x_2$ and $a \ggg_L x_1 = a \ggg_L x_2$; 3) $\forall x_1 \equiv x_2 \pmod{L}$, $(a \lll_L x_1) \ggg_L x_2 = (a \ggg_L x_1) \lll_L x_2 = a$. The proofs are simple, therefore omitted. These properties will be directly used hereinafter without further explanations.

Both encryption schemes work with L-bit blocks (some zero bits are padded when the last plain-block contains less than L bits). In [14] $L = 64$ and in [15] $L = 32$, so the two encryption schemes are 64-bit and 32-bit block ciphers, respectively. Throughout the paper, we assume that the plaintext contains N blocks: $\{P_j\}_{j=0}^{N-1}$, and the corresponding ciphertext is $\{C_j\}_{j=0}^{N-1}$.

2.1 Encryption Scheme Based on the Logistic Map [14]

In this scheme, *the secret key* is the initial condition $x(0)$ and control parameter μ of the following chaotic logistic map:

$$f(x) = \mu x(1 - x). \tag{1}$$

The core of the encryption scheme is a PRBS, $\{B_i\}_{i=0}^{70N-1}$, which is generated from the chaotic logistic map. Two pseudorandom number sequences (PRNS), $\{A_j\}_{j=0}^{N-1}$ and $\{D_j\}_{j=0}^{N-1}$, are further derived from the PRBS for the encryption/decryption purpose. The whole procedure can be described in the following steps[2].

- *Step 1*: Set $j = 0$, $r = 3$ and iterate the logistic map from $x(0)$ for $N_0 = 250$ times.
- *Step 2*: Iterate the logistic map for $N_1 = 70$ times to get a sequence composing of 70 chaotic states. Then, extract the r-th bit from each chaotic state's binary representation to get 70 pseudorandom bits $\{B_i\}_{i=70j}^{70j+69}$.
- *Step 3*: Set $A_j = \sum_{k=0}^{63} \left(B_{70j+k} \cdot 2^{63-k} \right)$, $D_j = \sum_{k=64}^{69} \left(B_{70j+k} \cdot 2^{69-k} \right)$ and $j = j + 1$.
- *Step 4*: If $j \leq N - 1$, iterate the logistic map for D_j times and then goto *Step 2*; otherwise, stop the process.

[2] To give a clearer and simpler description, we change some notations used in [14].

After the two PRNS $\{A_j\}_{j=0}^{N-1}$ and $\{D_j\}_{j=0}^{N-1}$ have been determined, *the encryption procedure* can be described easily by the following equation:

$$C_j = (P_j \lll_{64} D_j) \oplus A_j. \tag{2}$$

Accordingly, the decryption procedure is as follows:

$$P_j = (C_j \oplus A_j) \ggg_{64} D_j. \tag{3}$$

2.2 Encryption Scheme Based on a Delayed Chaotic Neural Network [15]

Compared with the scheme introduced in the last subsection, the DCNN-based one can be described as follows.

(1) The chaotic system is replaced by a DCNN with $n = 2$ neurons, described by the following equation:

$$\begin{pmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{pmatrix} = -\mathbf{C}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \mathbf{A}\begin{pmatrix} \tanh(x_1(t)) \\ \tanh(x_2(t)) \end{pmatrix} + \mathbf{B}\begin{pmatrix} \tanh(x_1(t-\tau(t))) \\ \tanh(x_2(t-\tau(t))) \end{pmatrix}, \tag{4}$$

where $(x_1(t), x_2(t))^T \in \mathbb{R}^2$ is the state vector associated with the 2 neurons, $\tau(t)$ is a time-delay function, $\mathbf{C} = \mathrm{diag}(c_1, c_2)$ is a diagonal matrix, $\mathbf{A} = [a_{i,j}]_{2\times 2}$, $\mathbf{B} = [b_{i,j}]_{2\times 2}$ are the connection weight matrix and the delayed weight matrix, respectively.

As the main cryptanalysis given in this paper does not depend on this chaotic neural network, more details about this n-dimensional chaotic system are referred to [15, Sec. 2]. Since the chaotic neural network is an analogue dynamical system, it has to be approximated by a discrete-time one by using a numerical algorithm with time step h.

(2) The secret key was claimed to include the initial condition and control parameters of the DCNN, the value of h, the structure of the DCNN and the numerical algorithm that implements the DCNN.

(3) One neuron of the DCNN is selected to generate a shorter PRBS, $\{B_i\}_{i=0}^{38N-1}$, for the encryption of each plain-block. The generation process of the PRBS is now changed as follows, where s is used to choose one neuron for encryption of the next plain-block.

- *Step 1*: Set $j = 0$, $r = 4$, $s = 1$, and iterate the DCNN from its initial condition for $N_0 = 1000$ time steps.
- *Step 2*: Iterate the DCNN for $N_1 = 38$ time steps. For each state of the s-th neuron, scale it to be within the unit interval $[0, 1]$ and then extract the r-th bit from the binary representation of the scaled state, so as to get 38 pseudorandom bits $\{B_i\}_{i=38j}^{38j+37}$.
- *Step 3*: Set $A_j = \sum_{k=0}^{31}\left(B_{38j+k} \cdot 2^{31-k}\right)$, $D_j = \sum_{k=32}^{36}\left(B_{38j+k} \cdot 2^{36-k}\right)$, $s = B_{38j+37} + 1$ and $j = j + 1$.

- *Step 4*: If $j \leq N-1$, iterate the DCNN for D_j time steps and then goto *Step 2*; otherwise, stop the process.

(4) An extra bit shift operation is introduced on A_j. By doing that, the encryption procedure becomes

$$C_j = (P_j \lll_{32} D_j) \oplus (A_j \ggg_{32} D_j). \tag{5}$$

Similarly, the decryption procedure is changed to

$$P_j = (C_j \oplus (A_j \ggg_{32} D_j)) \ggg_{32} D_j. \tag{6}$$

3 Some Security Problems

3.1 Insufficient Randomness of Chaos-Based PRBS $\{B_i\}$

In both encryption schemes, it is expected that the PRBS is random enough to ensure a high level of security. However, as shown below, neither the chaotic trajectories of the logistic map nor those of the DCNN have a uniform distribution, which leads to insufficient randomness of the PRBS generated from these chaotic trajectories.

For the logistic map, distributions of a number of chaotic trajectories, generated by iterating Eq. (1) for 10^5 times with random initial conditions and random control parameters, were studied. All the distributions are quite close to each other, so only one typical example is shown in Fig. 1 for illustration. Apparently, the non-uniform distribution of the chaotic trajectory will inevitably degrade the randomness of the derived PRBS $\{B_i\}$. For verification, we employed the NIST statistical test suite [17] to test the randomness of 100 binary sequences of length $\frac{256 \cdot 256}{8} \cdot 70 = 573440$ (the number of bits used for encryption of a 256×256 plain gray-scale image). Note that the 100 binary sequences were generated with randomly selected secret keys. For each test, the default significance level 0.01 was used. The results are shown in Table 1, from which one can see that the PRBS $\{B_i\}$ does not satisfy the requirements as a good random source.

For the DCNN used in [15], the non-uniformity of the chaotic trajectory corresponding to each neuron is even worse. Observing Fig. 1 in [15], one can easily see that the trajectory seldom visits some regions of the phase space. We performed some experiments to further investigate this problem [3] . Figure 2 shows

[3] In the experiments, the involved delay differential equation (DDE) with a time-varying delay was numerically solved by the method proposed in [18] with the same default error tolerance.

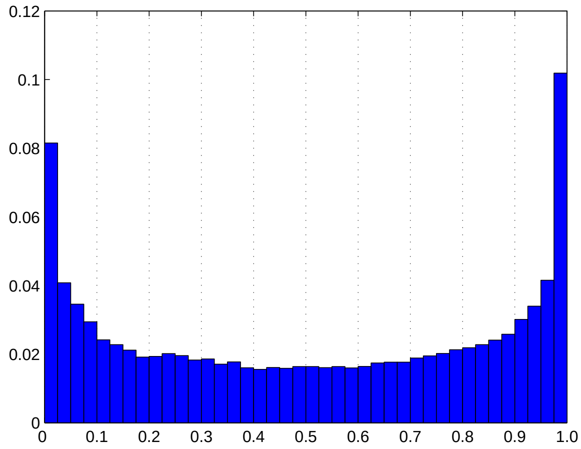


Fig. 1. A typical distribution of some random chaotic trajectories of the logistic map with control parameter $\mu = 3.999$.

Table 1
The performed tests and the number of sequences passing each test in a sample of 100 sequences.

Name of Test	Number of Passed Sequences
Frequency	0
Block Frequency	3
Cumulative Sums	0
Runs	0
Rank	82
Discrete Fourier Transform	32
Non-overlapping Template Matching	0
Serial	0
Approximate Entropy	0

the distributions of 622,600 chaotic states of the two neurons of the DCNN with the following configurations: the initial condition $\boldsymbol{x}(t \leq 0) = (0.4, 0.6)^T$, $\tau(t) = 1 + 0.1\sin(t)$, and the three matrices in Eq. (4) were set as follows:

$$\mathbf{A} = \begin{pmatrix} 2 & -0.1 \\ -5 & 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The time step size $h = 0.01$ was used in the numerical solution to simulate the DCNN.

The distributions shown in Fig. 2 imply that the randomness of the PRBS

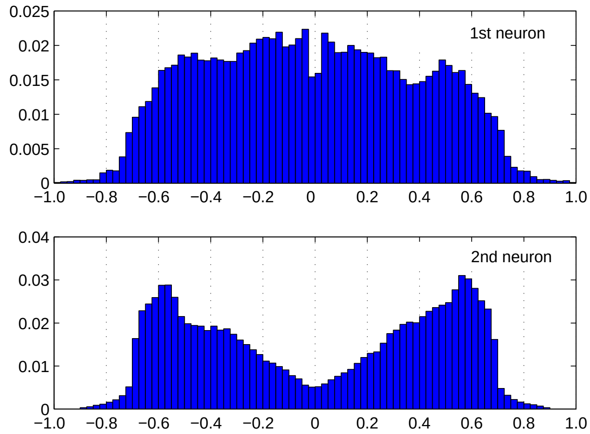


Fig. 2. The empirical distributions of the two neurons' states of the DCNN Eq. (4).

$\{B_i\}$ is weaker than that derived from the logistic map. Moreover, there exists another more serious problem that dramatically influences the randomness of the PRBS derived from the DCNN. As can be seen, the DCNN is an analogue dynamical system with a continuous trajectory, which means that any two consecutive chaotic states simulated via a numerical algorithm are always closely correlated. As a result, the chaotic bits derived from consecutive chaotic states will also be closely correlated. Furthermore, the smaller the time step size h is, the stronger such a correlation will be. However, as mentioned in the last subsection, the time step size h should be small enough to achieve a good estimation of the true dynamics of the DCNN. That is, the close correlation between consecutive bits is an unavoidable defect of PRBS based on any analogue dynamical system like this DCNN.

To evaluate the real randomness of the PRBS $\{B_i\}$ derived from the DCNN, we carried out the runs test [19, Sec. 5.4.4] on the first 20,000 bits of $\{B_i\}$ corresponding to the trajectory shown in Fig. 2, where the definition of run of a binary sequence is given in [17, Sec. 2.3.1]: "*A run of length k consists of exactly k identical bits and is bounded before and after with a bit of the opposite value*". The result is shown in Fig. 3. As a comparison, the mathematical expectations of the number of runs of various lengths in an ideal random binary sequence are also plotted. Observing Fig. 3, one can see that the randomness of the PRBS generated by the DCNN is obviously very weak. As a result of the poor randomness of $\{B_i\}$, it is expected that $\{A_j\}$ and $\{D_j\}$ are also far from being random, which can be clearly seen by looking at the numbers of different values in $\{\lfloor A_j/2^{22}\rfloor\}_{j=0}^{16383}$ and $\{D_j\}_{j=0}^{16383}$, as shown in Fig. 4.

Due to the serious non-uniform distributions of $\{A_j\}$ and $\{D_j\}$ in the DCNN-based scheme, it is suspected that the encryption performance may not be satisfactory. For example, Fig. 4 shows that the probability that $A_j = D_j = 0$ is relatively high, which means that the encryption totally fails in this case.

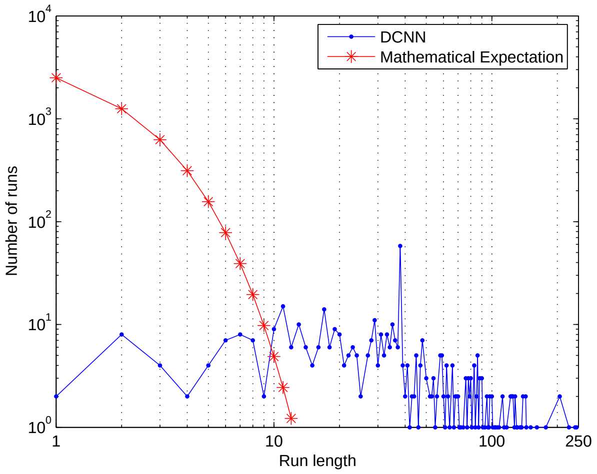


Fig. 3. The numbers of k-bit runs in one PRBS $\{B_i\}_{i=0}^{19999}$ generated by the DCNN versus the expected numbers of a random bit sequence, where $k = 1 \sim 250$.

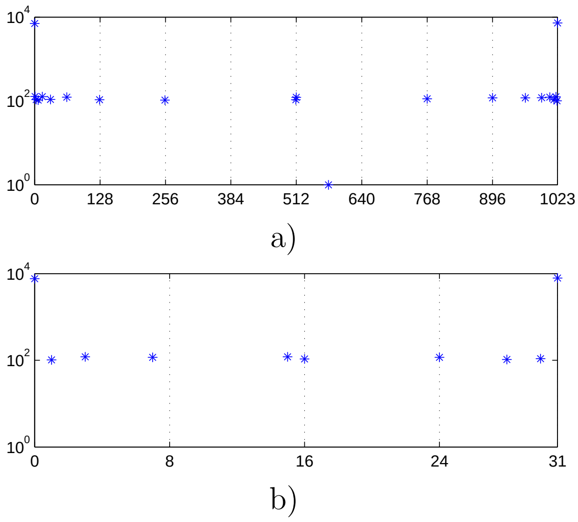
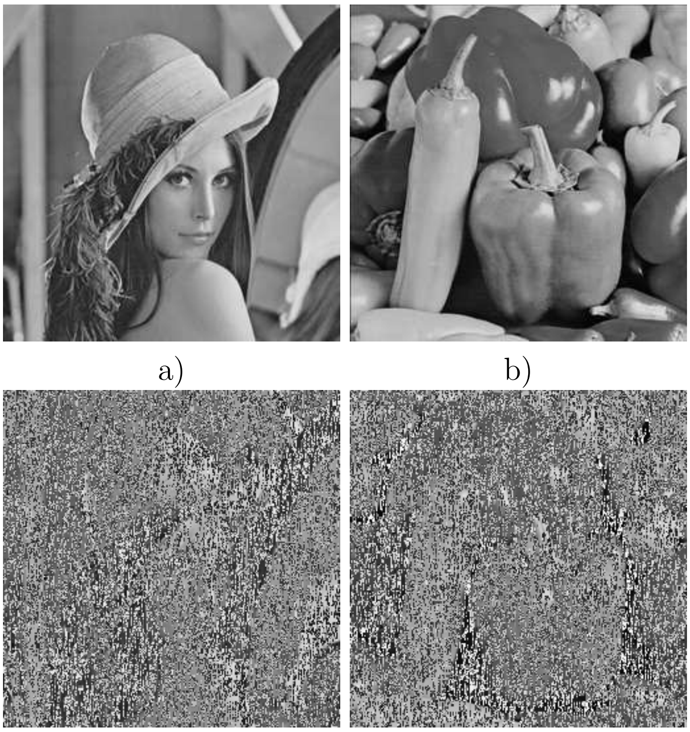


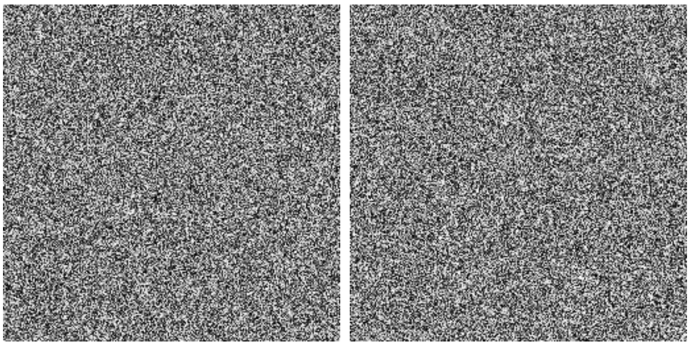
Fig. 4. The numbers of different values of a) $\left\{\left\lfloor A_j/2^{22}\right\rfloor\right\}_{j=0}^{16383}$ and b) $\{D_j\}_{j=0}^{16383}$. Note that only the numbers of values existing in the sequences are plotted in the two sub-figures.

For two typical images, "Lenna" and "Peppers", the encryption results of the DCNN-based scheme are shown in Fig. 5, where the secret key was set to be the one used for Fig. 2. One can see that some visual information about the plain-images has been leaked from the cipher-images. As a comparison, the encryption results of the scheme based on the logistic map is given in Fig. 6, where the original secret key in [14] was used: $x(0) = 0.1777$, $\mu = 3.9999995$. Comparing Figs. 5 and 6, one can see that the DCNN-based scheme has a much worse encryption performance than the one based on the logistic map.



a) b) c) d)

Fig. 5. Encryption results of the DCNN-based encryption scheme on two typical plain-images: a) the plain-image “Lenna”; b) the plain-image “Peppers”; c) the cipher-image of “Lenna”; d) the cipher-image of “Peppers”.



a) b)

Fig. 6. The encryption results of the encryption scheme based on the logistic map on the two plain-images “Lenna” and “Peppers”: a) the cipher-image of “Lenna”; b) the cipher-image of “Peppers”.

3.2 Some Inadequate Sub-Keys in the DCNN-Based Scheme

In [15], it was stated that the transfer/time-delay functions of each neuron and the numerical algorithm itself are also part of the secret key. However, these algorithmic details are generally embedded in the codes of the encryption/decryption machines, so they can be reversely engineered by analyzing the encryption/decryption machines. As a result, they are not suitable as part of the secret key to ensure the security of the designed cryptosystem [20, Sec. 1.1.7].

Of course, if a number of candidate algorithms are embedded in the cryptosystem, a sub-key may be introduced to secretly choose one for encryption and decryption. With such a measure, the size of the sub-key space is limited to the number of the candidate algorithms, which is not large enough to make the cryptosystem feasible in practice.

There is another problem with other sub-keys in this scheme. When the structure of the chaotic neural network is fixed, some limits have to be exerted on the values of the control parameters to ensure the chaoticity of the dynamical system [21]. For the time step size of the numerical algorithm, this problem also exists, as the time step size must be small enough to approach the true dynamics of the DCNN. This problem will reduce the key space of the encryption scheme to some extent.

3.3 Low Sensitivity of Encryption to Plaintexts

As was well-known in cryptography [5], a good cryptosystem should be sufficiently sensitive to small changes in the plaintext. However, this property does not hold for the two encryption schemes proposed in [14, 15]. Observing Eqs. (2) and (5), it is clear that one bit change in P_j will cause only one bit change in C_j, in the case that the same secret key is used (i.e., both A_j and D_j are the same). If there are two different bits with different values, the distance between them modulo L will also remain unchanged after encryption.

4 Cryptanalysis

To facilitate the cryptanalysis given in this section, the encryption processes of the two encryption schemes are first unified as follows:

$$C_j = (P_j \lll_L D_j) \oplus A'_j, \tag{7}$$

where $A'_j = A_j$ with $L = 64$ for the scheme in [14] and $A'_j = (A_j \ggg_L D_j)$ with $L = 32$ for the scheme in [15]. Apparently, if the two sequences $\{D_j\}_{j=0}^{N-1}$ and $\{A'_j\}_{j=0}^{N-1}$ can be reconstructed, then they can be used as an equivalent key to decrypt the N leading plain-blocks of any plaintext that is encrypted with the same key, as follows:

$$P_j = (C_j \oplus A'_j) \ggg_L D_j. \tag{8}$$

This can be done by employing some special properties of circular bit shift operations in known/chosen-plaintext attacks. Because this idea of cryptanalysis is completely independent of the underlying chaotic systems, it can work well for both schemes.

In the first part of this section, we give some properties of the circular operations and then show how two very efficient and successful attacks can be developed based on these properties.

4.1 Some Special Properties of Circular Bit Shift Operations

The circular bit shift operations have the following properties[4].

Property 1 *Assume that* $L, \tau \in \mathbb{Z}^+$, $x \in \mathbb{Z}$, $a^* \in \{0, \cdots, 2^\tau - 1\}$ *and* $\tau \mid L$. *If* $a = \sum_{i=0}^{L/\tau-1} (a^* \cdot 2^{\tau i})$ *and* $x \equiv 0 \pmod{\tau}$, *then* $(a \lll_L x) = (a \ggg_L x) = a$.

Proof: This property is a direct consequence of the definitions of L-bit left and right circular bit shift operations. ■

Property 2 *Assume that* $L \in \mathbb{Z}^+$, $x \in \mathbb{Z}$ *and* $a, b \in \{0, \cdots, 2^L - 1\}$. *Then,* $(a \lll_L x) \oplus (b \lll_L x) = (a \oplus b) \lll_L x$ *and* $(a \ggg_L x) \oplus (b \ggg_L x) = (a \oplus b) \ggg_L x$.

Proof: Assume that $a = \sum_{i=0}^{L-1} (a_i \cdot 2^i)$ and $b = \sum_{i=0}^{L-1} (b_i \cdot 2^i)$. Then, $(a \lll_L x) \oplus (b \lll_L x) = \left(\sum_{i=0}^{L-1} \left(a_i \cdot 2^{(i+x) \bmod L}\right)\right) \oplus \left(\sum_{i=0}^{L-1} \left(b_i \cdot 2^{(i+x) \bmod L}\right)\right) = \sum_{i=0}^{L-1} (a_i \oplus b_i) \cdot 2^{(i+x) \bmod L)} = (a \oplus b) \lll_L x$. In a similar process, $(a \ggg_L x) \oplus (b \ggg_L x) = (a \oplus b) \ggg_L x$ can also be proved. ■

Property 3 *Assume that* $L \in \mathbb{Z}^+ \backslash \{1\}$, *and* $a = \sum_{i=0}^{L-1} (a_i \cdot 2^i) \in \{0, \cdots, 2^L - 1\}$, *where* $a_i \in \{0, 1\}$. *If there exists* $x \in \{1, \cdots, L-1\}$ *such that* $a \lll_L x = a$, *then there must exist* $\tau \mid \gcd(L, x)$ *and* $a^* \in \{0, \cdots, 2^\tau - 1\}$ *such that* $a = \sum_{i=0}^{L/\tau-1} (a^* \cdot 2^{\tau i})$.

Proof: This property is proved via mathematical induction on x.

When $x = 1$, the condition $a \lll_L 1 = a$ means the following: $a_0 = a_1$, $a_1 = a_2$, $\cdots$, $a_{L-1} = a_0$, which immediately leads to the result that $a_0 = a_1 = \cdots = a_{L-1}$. Then, setting $\tau = 1$ and $a^* = a_0 = \cdots = a_{L-1}$, one has $a = \sum_{i=0}^{L-1} (a^* \cdot 2^i) = \sum_{i=0}^{L/\tau-1} (b \cdot 2^{\tau i})$, where $(\tau = 1) \mid \gcd(L, x)$.

[4] Property 1 has already been pointed out in [6, Sec. 7.4.1] for another image encryption scheme based on the same operations.

Now, assume the property is true for all integers smaller than $x \geq 2$. We will prove that it also holds for x. Consider two different conditions as follows.

C1) When $x \mid L$: from the condition $a \lll_L x = a$, it follows that a can be divided into L/x identical segments, each of which has x bits. Setting $\tau = x$ and $a^* = \sum_{i=0}^{x-1}(a_i \cdot 2^i)$, we have $a = \sum_{i=0}^{L/x-1}(a^* \cdot 2^{xi}) = \sum_{i=0}^{L/\tau-1}(a^* \cdot 2^{\tau i})$, where $(\tau = x) \mid (\gcd(L, x) = x)$.

C2) When $x \nmid L$: divide all the L bits into $\lceil L/x \rceil$ bit segments, among which the last one contains only $\hat{x} = (L \bmod x)$ bits. That is, a can be represented as $A \cdots A\hat{A}$, where $A = a_0 \cdots a_{x-1}$ and $\hat{A} = \hat{a}_0 \cdots \hat{a}_{\hat{x}-1}$. Performing $a \lll_L x$ and comparing it with a (note that $a \lll_L x = a$), one can get $\forall i = 0 \sim (\hat{x} - 1)$, $\hat{a}_i = a_i$. Thus, a becomes $\hat{A}\check{A} \cdots \hat{A}\check{A}\hat{A}$, where $\hat{A} = a_0 \cdots a_{\hat{x}-1}$, $\check{A} = a_{\hat{x}} \cdots a_{x-1}$ and $A = \hat{A}\check{A}$. Then, performing $a \lll_L x$ and comparing it with a again, one has $\check{A}\hat{A} = \hat{A}\check{A} = A$. This means that $A \lll_x \hat{x} = A$. Since $\hat{x} < x$, by the assumption of the mathematical induction, there exists $\tau \in \mathbb{Z}$ such that $A = \sum_{i=0}^{x/\tau-1}(a^* \cdot 2^{\tau i})$, where $\tau \mid \gcd(x, \hat{x})$ and $a^* \in \{0, \cdots, 2^\tau - 1\}$. Since $\tau \mid \hat{x}$ also holds, one has $\hat{A} = \sum_{i=0}^{\hat{x}/\tau-1}(a^* \cdot 2^{\tau i})$. Finally, therefore, $a = \sum_{i=0}^{L/\tau-1}(a^* \cdot 2^{\tau i})$.

The above induction completes the proof of the property. ■

Remark 1 *In Property 3, if one changes $a \lll_L x = a$ to another form, $a = a \lll_L (L - x)$, the condition of τ will become $\tau \mid \gcd(L, L - x)$. This is actually equivalent to $\tau \mid \gcd(L, x)$, as $\gcd(L, x) = \gcd(L, L - x)$.*

Combining Properties 1 and 3, one can easily derive the following theorem.

Theorem 1 *Assume that $L \in \mathbb{Z}^+ \backslash \{1\}$, $x \in \mathbb{Z}$ and $a, b \in \{0, \cdots, 2^L - 1\}$. The equation $(a \lll_L x) = b$ (x is the unknown) has more than one solution modulo L if and only if there exists $\tau < L$, $\tau \mid L$ and $a^* \in \{0, \cdots, 2^\tau - 1\}$ such that $a = \sum_{i=0}^{L/\tau-1}(a^* \cdot 2^{\tau i})$.*

Proof: The "if" and "only if" parts of this theorem are direct consequences of Properties 1 and 3, respectively. ■

An alternative form of Theorem 1 is as follows.

Theorem 2 *Assume that $L \in \mathbb{Z}^+ \backslash \{1\}$, $x \in \mathbb{Z}$ and $a, b \in \{0, \cdots, 2^L - 1\}$. The equation $(a \lll_L x) = b$ (x is the unknown) has only one solution modulo L if and only if there does not exist $\tau < L$, $\tau \mid L$ and $a^* \in \{0, \cdots, 2^\tau - 1\}$ satisfying $a = \sum_{i=0}^{L/\tau-1}(a^* \cdot 2^{\tau i})$.*

When $\tau < L$, $a = \sum_{i=0}^{L/\tau-1}(a^* \cdot 2^{\tau i})$ actually means that a can be represented by repeated bit patterns. For example, when $L = 8$, $\tau = 4$ and $a^* = (1001)_2 = 9$, one has $a = (10011001)_2 = 153$, where $(\cdots)_2$ denotes the binary representation (the same below).

4.2 Chosen-Plaintext Attack

In this attack, two plaintexts can be deliberately chosen to ensure that all elements in $\{D_j\}$ and $\{A'_j\}$ are uniquely determined. By choosing a plaintext such that $P_j^{(1)} = 0$ or $2^L - 1$, $\forall j = 0 \sim (N-1)$, one obtains $(P_j^{(1)} \lll_L D_j) = P_j^{(1)}$ and further gets $A'_j = P_j^{(1)} \oplus C_j^{(1)}$. After recovering $\{A'_j\}_{j=0}^{N-1}$, one may choose another plaintext such that each $P_j^{(2)}$ cannot be represented by repeated bit patterns, for example, $P_j^{(2)} = 152 = (10011000)_2$ when $L = 8$. Then, by Theorem 2, the value of D_j can always be uniquely determined by solving the following equation:

$$(P_j^{(2)} \lll_L D_j) = C_j^{(2)} \oplus A'_j.$$

4.3 Differential Known-Plaintext Attack

When the same key is used to encrypt two plaintexts, $\{P_j^{(1)}\}_{j=0}^{N-1}$ and $\{P_j^{(2)}\}_{j=0}^{N-1}$, using Eq. (7) and Property 2 one can easily deduce the following equality:

$$\begin{aligned} C_j^{(1)} \oplus C_j^{(2)} &= \left(P_j^{(1)} \lll_L D_j\right) \oplus \left(P_j^{(2)} \lll_L D_j\right) \\ &= \left(P_j^{(1)} \oplus P_j^{(2)}\right) \lll_L D_j. \end{aligned} \tag{9}$$

The above equation means that $\{A'_j\}_{j=0}^{N-1}$ are completely circumvented in a differential attack. Then, one can try to determine the value of D_j by searching all L possible values. From Theorem 2, the value D_j can be uniquely determined if $P_j^{(1)} \oplus P_j^{(2)}$ cannot be represented in repeated bit patterns. After obtaining D_j, one can further get the value of A'_j as follows:

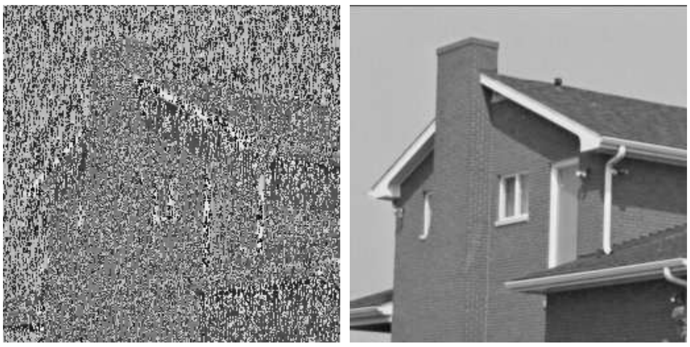
$$A'_j = \left(P_j^{(1)} \lll_L D_j\right) \oplus C_j^{(1)}. \tag{10}$$

Now, let us find the probability that the value of each D_j cannot be uniquely determined by solving Eq. (9), i.e., the probability that $P_j^{(1)} \oplus P_j^{(2)}$ can be represented as repeated bit patterns. Under the assumptions that $P_j^{(1)} \oplus P_j^{(2)}$ has a uniform distribution over $\{0, \cdots, 2^L - 1\}$ and that any two differential values are independent of each other, this probability can be calculated to be

$$p = \frac{\sum_{\tau < L,\, \tau | L} 2^\tau}{2^L}. \tag{11}$$

Then, it can be easily calculated that $p \approx 2^{-16}$ when $L = 32$ and $p \approx 2^{-32}$ when $L = 64$. In practice, this probability is generally larger than the theoretical value due to the non-uniform distribution of the plaintext and the correlation existing in two differential plaintexts. To the advantage of the attacker, our experiments have shown that this probability is still very small in most cases. The small probability ensures that it is a high-probability event to uniquely determine the value of D_j with only two known plaintexts and their corresponding ciphertexts.

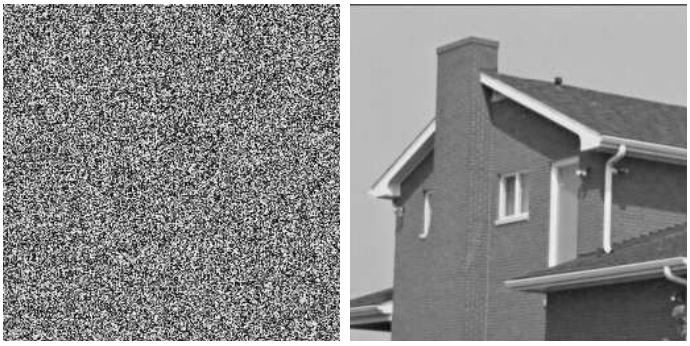
To evaluate the performance of the differential plaintext attacks, some experiments were carried out when a number of natural images are chosen as plaintexts. Consider the case of the 2-neuron DCNN shown in Eq. (4) with the same configurations set in Sec. 3.1. With the two plain-images and the corresponding cipher-images shown in Fig. 5, we reconstructed $\{D_j\}_{j=0}^{16383}$ and $\{A'_j\}_{j=0}^{16383}$. In all the 16384 elements of each sequence, only two 1's could not be uniquely determined, which is about 0.012% ($\approx 2^{-13}$). Then, the two reconstructed sequences $\{D_j\}_{j=0}^{16383}$ and $\{A'_j\}_{j=0}^{16383}$ were used to decrypt a cipher-image shown in Fig. 7a (which corresponds to a plain-image "House"). The recovered plain-image is given in Fig. 7b. One can see that the breaking performance is nearly perfect.



a) b)

Fig. 7. A near-perfect breaking result of the differential known-plaintext attack on the DCNN-based scheme: a) the cipher-image corresponding to a plain-image "House"; b) the decrypted plain-image.

The same experiments were also carried out on the scheme based on the logistic map with the same known plain-images and the corresponding cipher-images shown in Fig. 6. As analyzed above, in this case the probability that each value of D_j and A'_j cannot be uniquely determined is estimated to be 2^{-32}. Considering there are only $256 \times 256/8 = 2^{13}$ plain-blocks, one can expect that all elements in $\{D_j\}_{j=0}^{8191}$ and $\{A'_j\}_{j=0}^{8191}$ will be uniquely determined in very high probability, thus leading to a perfect breaking of the plain-image. Our experiments well agreed with this expectation. Figure 8 shows the breaking result on the plain-image "House".



a) b)

Fig. 8. A perfect breaking result of the differential known-plaintext attack on the encryption scheme based on the logistic map: a) the cipher-image corresponding to “House”; b) the recovered plain-image.

5 Conclusions

This paper has analyzed the security of two chaotic encryption schemes based on circular bit shift and XOR operations. It has been found that these two schemes are insecure against the differential known-plaintext attack and the chosen-plaintext attack, in which only two known/chosen plaintexts are required to achieve a perfect breaking performance. Moreover, some other security problems existing in the two encryption schemes have been pointed out. Our cryptanalytic results suggest that the two encryption schemes should be further enhanced before they can be used in applications requiring a high level of security.

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References

[1] L. Kocarev, G. Jakimoski, T. Stojanovski, U. Parlitz, From chaotic maps to encryption schemes, in: Proc. IEEE Int. Symposium Circuits and Systems 98, Vol. 4, IEEE, 1998, pp. 514–517.

[2] G. Alvarez, F. Montoya, G. Pastor, M. Romera, Chaotic cryptosystems, in: Proc. IEEE Int. Carnahan Conf. Security Technology, IEEE, 1999, pp. 332–338.

[3] C. P. Silva, A. M. Young, Introduction to chaos-based communications and signal processing, in: Proc. IEEE Aerospace Conf., IEEE, 2000, pp. 279–299.

[4] L. Kocarev, Chaos-based cryptography: A brief overview, IEEE Circuits and Systems Maganize 1 (3) (2001) 6–21.

[5] R. Schmitz, Use of chaotic dynamical systems in cryptography, J. Franklin Institute 338 (4) (2001) 429–441.

[6] S. Li, Analyses and new designs of digital chaotic ciphers, Ph.D. thesis, School of Electronics and Information Engineering, Xi'an Jiaotong University, Xi'an, China, available online at `http://www.hooklee.com/pub.html` (2003).

[7] S. Li, G. Chen, X. Zheng, Chaos-based encryption for digital images and videos, in: B. Furht, D. Kirovski (Eds.), Multimedia Security Handbook, CRC Press, 2004, Ch. 4, pp. 133–167, preprint available at `http://www.hooklee.com/pub.html`.

[8] C. Li, Cryptanalyses of some multimedia encryption schemes, Master's thesis, Department of Mathematics, Zhejiang University, Hangzhou, China, available online at `http://eprint.iacr.org/2006/340` (2005).

[9] S. Li, Digital chaotic ciphers, invited talk at the Department of Physics, Beijing Normal University, Beijing, China, slides available at `http://www.hooklee.com/Talks/CC2.pdf` (2005).

[10] M. Hasler, Synchronization of chaotic systems and transmission of information, Int. J. Bifurcation Chaos 8 (4) (1998) 647–659.

[11] T. Yang, A survey of chaotic secure communication systems, International Journal of Computational Cognition 2 (2) (2004) 81–130.

[12] S. Li, Analog chaos-based secure communications, invited talk at the College of Information Technology, Shenzhen University, Shenzhen, China, slides available at `http://www.hooklee.com/Talks/CC1b.pdf` (2005).

[13] G. Alvarez, S. Li, Some basic cryptographic requirements for chaos-based cryptosystems, Int. J. Bifurcation Chaos 16 (8) (2006) 2129–2151.

[14] T. Xiang, X. Liao, G. Tang, Y. Chen, K. Wong, A novel block cryptosystem based on iterating a chaotic map, Physics Letters A 349 (1-4) (2006) 109–115.

[15] W. Yu, J. Cao, Cryptography based on delayed chaotic neural networks, Physics Letters A 356 (4-5) (2006) 333–338.

[16] A. Palacios, H. Juarez, Cryptography with cycling chaos, Physics Letters A 303 (5-6) (2002) 345–351.

[17] A. Rukhin, et al., A statistical test suite for random and pseudorandom number generators for cryptographic applications, NIST Special Publication 800-22, available online at `http://csrc.nist.gov/rng/rng2.html` (2001).

[18] L. Shampine, Solving ODEs and DDEs with residual control, online document, `http://faculty.smu.edu/lshampin/residuals.pdf`, matlab codes available at `http://faculty.smu.edu/lshampin/ddesd.zip` (2006).

[19] A. Menezes, P. van Oorschot, S. Vanstone, Handbook of Applied Cryptography, CRC Press, 1996.

[20] B. Schneier, Applied Cryptography - Protocols, Algorithms, and Souce Code in C, 2nd Edition, John Wiley & Sons, Inc., New York, 1996.

[21] H. Lu, Chaotic attractors in delayed neural networks, Physics Letters A 298 (2-3) (2002) 109–116.