Return-Map Cryptanalysis Revisited*

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Abstract

As a powerful cryptanalysis tool, the method of return-map attacks can be used to extract secret messages masked by chaos in secure communication schemes. Recently, a simple defensive mechanism was presented to enhance the security of chaotic parameter modulation schemes against return-map attacks. Two techniques are combined in the proposed defensive mechanism: multistep parameter modulation and alternative driving of two different transmitter variables. This paper re-studies the security of this proposed defensive mechanism against return-map attacks, and points out that the security was much over-estimated in the original publication for both ciphertext-only attack and known/chosen-plaintext attacks. It is found that a deterministic relationship exists between the shape of the return map and the modulated parameter, and that such a relationship can be used to dramatically enhance return-map attacks thereby making them quite easy to break the defensive mechanism.

1 Introduction

In the past two decades, chaotic systems have been extensively used to construct cryptosystems in either analog [Álvarez *et al.*, 1999; Yang, 2004] or digital [Li, 2003] forms. Most analog implementations are secure communication systems based on synchronization of the sender and the receiver chaotic systems [Pecora & Carroll, 1990], where the signal is transmitted over a public channel from the sender to drive the receiver for achieving synchronization and message decryption. Some different encryption structures have been proposed: chaotic masking [Kocarev *et al.*, 1992; Murali & Lakshmanan, 1994; Morgul & Feki, 1999], chaotic switching or chaotic shift keying (CSK) [Parlitz *et al.*, 1992; Dedieu *et al.*, 1993; Parlitz & Ergezinger, 1994], chaotic modulation [Wu & Chua, 1993; Yang & Chua, 1996; Parlitz *et al.*, 1996], and the inverse system approach [Feldmann *et al.*, 1996]. At the same time, many different cryptanalysis methods have also been developed to break the proposed chaos-based secure communication systems: return-map attacks [Pérez & Cerdeira, 1995; Zhou & Chen, 1997; Yang *et al.*, 1998c; Li *et al.*, 2005b], nonlinear prediction based attacks [Short, 1994, 1997; Zhou & Lai, 1999], spectral analysis attacks [Yang *et al.*, 1998a; Álvarez & Li, 2004a], generalized synchronization (GS) based attacks [Yang *et al.*, 1998b; Álvarez *et al.*, 2005a, 2004b], short-time period based attacks [Yang, 1995; Álvarez & Li, 2004b], parameter identification based attacks [Stojanovski *et al.*, 1996; Tao *et al.*, 2003; Vaidya & Angadi, 2003; Álvarez *et al.*, 2004a], and so on.

Given the existence of so many different attacks, it has become a real challenge to design highly secure chaos-based communication systems against all known attacks. Three general countermeasures have been proposed in the literature: 1) using more complex dynamical systems, such as high-dimensional hyperchaotic systems or multiple cascaded (heterogeneous) chaotic systems [Grassi & Mascolo, 1999a; Murali, 2000; Yao *et al.*, 2003]; 2) introducing traditional ciphers into the chaotic cryptosystems [Yang *et al.*, 1997; Grassi & Mascolo, 1999b; Lian *et al.*, 2003]; 3) introducing an impulsive (also named sporadic) driving signal instead of a continuous signal to realize synchronization [Yang & Chua, 1997; He *et al.*, 2000; Khadra *et al.*, 2003]. The first countermeasure has been found insecure against some attacks [Short & Parker, 1998; Zhou & Lai, 1999; Huang *et al.*, 2001; Tao *et al.*, 2003], and some security defects of the second countermeasure have also been reported [Parker & Short, 2001], but the last one has not yet been cryptanalyzed to date.

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Besides the above three general countermeasures, there also exist some specially-designed countermeasures that can be used to resist certain attacks. This paper studies two such countermeasures, recently proposed by Palaniyandi & Lakshmanan [2001], against return-map attacks. These two proposed countermeasures are multistep parameter modulation and alternative driving of transmitter variables, which have been combined to construct a new secure communication scheme for binary signal transmissions. After refining return-map attacks via a deterministic relationship between the return map and a parameter b_s, we found that the security of the first countermeasure was much over-estimated in [Palaniyandi & Lakshmanan, 2001], and that the combination of the two countermeasures can be easily separated in some way so as to disable the second countermeasure. The aforementioned deterministic relationship between the return map and the parameter b_s is reported in this paper, for the first time in the literature, which is useful not only for engineering studies on chaos-based secure communications but also for theoretical studies on the dynamics of chaotic systems.

The rest of this paper is organized as follows. In the next section, a brief introduction to return-map attacks and related countermeasures is given. Section 3 re-evaluates the security of the multistep parameter modulation scheme, by exploiting a deterministic relationship between the shape of the return map and the modulated parameter b_s. The original return-map attack proposed by Pérez & Cerdeira [1995] will be enhanced. In Sec. 4, cryptanalysis of the scheme of alternative driving of transmitter variables is studied in detail. The last section concludes the paper.

2 Return-Map Attacks and Related Countermeasures

The return-map attack method was first proposed by Pérez & Cerdeira [1995] to break chaotic switching (binary parameter modulation) and chaotic masking schemes based on the Lorenz system, which was then studied by Yang *et al.* [1998c] to break chaotic masking, switching and non-autonomous modulation schemes based on Chua's circuit. In [Zhou & Chen, 1997], the return-map attack method was also used to break a DCSK scheme based on a discrete-time chaotic map [Parlitz & Ergezinger, 1994]. Without loss of generality, this paper will focus on the attack scheme of Pérez & Cerdeira on the Lorenz system thereby demonstrating how the return map is constructed and how the attack works to break a typical chaotic switching scheme proposed in [Cuomo & Openheim, 1993].

Consider the following Lorenz system used as the sender:

$$\begin{aligned} \dot{x}_s &= \sigma(y_s - x_s), \\ \dot{y}_s &= r_s x_s - y_s - x_s z_s, \\ \dot{z}_s &= x_s y_s - b_s z_s, \end{aligned} \tag{1}$$

where σ, b_s, r are system parameters, and the value of b_s is modulated by $m(t)$, the digital plain-signal for secure transmission, as follows:

$$b_s = \begin{cases} b_0, & m(t) = 0, \\ b_1, & m(t) = 1. \end{cases}$$

To transmit $m(t)$ to the receiver end, a variable of the sender system, such as x_s, is sent out, which will be used to induce synchronization of the receiver system, resulting in:

$$\begin{aligned} \dot{x}_r &= \sigma(y_r - x_r), \\ \dot{y}_r &= r_r x_s - y_r - x_s z_r, \\ \dot{z}_r &= x_s y_r - b_r z_r, \end{aligned} \tag{2}$$

where $b_r = b_0$. When $m(t) = 0$, the intended synchronization can be reached, while when $m(t) = 1$, the synchronization error always remains at a certain finite order. Then, it is easy to decode the secret signal $m(t)$ by checking the power energy $(x_r - x_s)^2$ with a digital filter. Following Cuomo & Openheim [1993], the parameters are set as $\sigma = 16$, $r = 45.6$, $b_0 = 4.0$ and $b_1 = 4.4$.

However, the above chaotic switching scheme can be easily broken with the return map constructed from x_s as pointed out in [Pérez & Cerdeira, 1995]. Assuming that X_m and Y_m are the m-th maxima and m-th minima of x_s, respectively, define the following four variables: $A_m = \frac{X_m+Y_m}{2}$, $B_m = X_m - Y_m$, $C_m = \frac{X_{m+1}+Y_m}{2}$, $D_m = Y_m - X_{m+1}$, and then construct two return maps, (A_m vs B_m) and ($-C_m$ vs $-D_m$), as shown in Fig. 1. The two maps are actually equivalent to each other, so we only consider the map (A_m vs B_m) in this paper. Note that there are three segments in the return map, and each segment is further split into two strips. It is obvious that the split of the map is caused by the switching of the value of b_s between b_0 and b_1. Thus, by checking which strip the point (A_m, B_m) falls on, one can easily unmask the current value of the digital signal $m(t)$. Since one has to assign either 0-bit or 1-bit to a strip in each segment, it was claimed in [Pérez

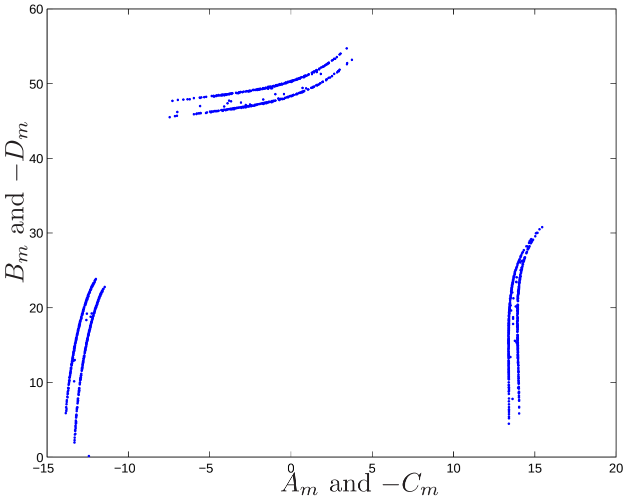


Figure 1: The return maps constructed for a typical chaotic switching scheme.

& Cerdeira, 1995] that there are only seven chances to make wrong assignments, which can be easily detected by observing the waveform of the reconstructed digital signal $m(t)$.

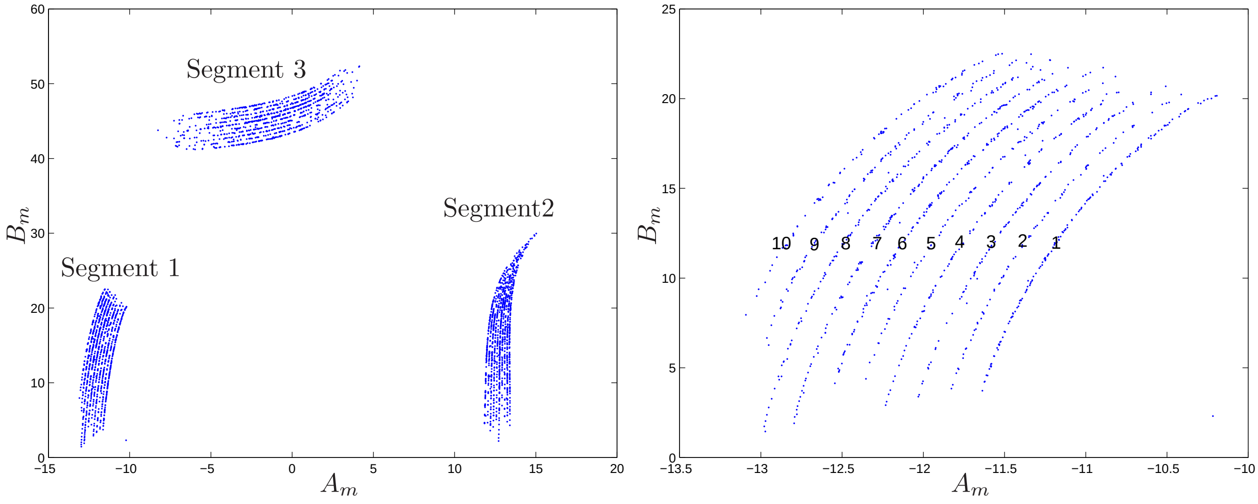
In recent years, some different countermeasures have been proposed to resist the above return-map attack. In [Bu & Wang, 2004], a periodic signal $g_0(t) = A\cos(\omega t + \phi_0)$ is combined with z_s to modulate the transmitted signal x_s so as to blur the reconstructed return map in order to frustrate the attacker. However, soon it was broken as reported in [Chee *et al.*, 2004; Wu *et al.*, 2004; Álvarez *et al.*, 2005b] via distinguishing the parameters ω, ϕ_0 and removing the modulating signal. A modified scheme of the original method of Bu & Wang [2004] was proposed in [Wu *et al.*, 2004] to further improve its security. Our recent work shows that this modified modulating scheme is still not secure enough [Li *et al.*, 2005a] and that the modulating signal can still be effectively removed via parameters estimation.

In [Palaniyandi & Lakshmanan, 2001], two new countermeasures were proposed and combined to enhance the security of chaotic switching schemes against return-map attacks. The first countermeasure is to increase the number of strips by modulating b_s between $2n$ different values: $b_{0,1}, \cdots, b_{0,n}$ and $b_{1,1}, \cdots, b_{1,n}$, where $b_{0,1}, \cdots, b_{0,n}$ correspond to $m(t) = 0$ and $b_{1,1}, \cdots, b_{1,n}$ correspond to $m(t) = 1$. This countermeasure is called *multistep parameter modulation*, and accordingly the original two-valued chaotic switching scheme is called *single-step parameter modulation*. It was claimed that the chances to make wrong assignments become $(2^{2n} - 2)^3 - 1 \approx 2^{6n}$ and that the security against return-map attacks is dramatically enhanced even when n is not too large. Figure 2 shows the return map constructed from x_s when the multistep parameter modulation is used, where $n = 5$ and $b_{0,i} \in \{3.2, 3.4, 3.6, 3.8, 4.0\}$, $b_{1,i} \in \{3.1, 3.3, 3.5, 3.7, 3.9\}$. It can be seen that each segment is split into $2n = 10$ strips. The second countermeasure is to alternatively use x_s and y_s as the driving signal to force the receiver system to synchronize with the sender, which will further split the constructed return map into two parts: one corresponds to the map from x_s and another to the map from y_s, as shown in Fig. 3[1]. It can be seen that two segments of the x_s-map and the y_s-map largely overlap each other. In a multistep parameter modulation system, the receiver contains $2n$ different driven sub-systems, which are used to realize synchronization for the $2n$ different values of b_s, respectively. When alternative driving is also applied, the number of sub-systems is doubled to be $4n$, among which $2n$ correspond to x_s-driving synchronization and another $2n$ to y_s-driving synchronization. For more details about the two countermeasures, see [Palaniyandi & Lakshmanan, 2001].

3 Re-Evaluating the Security of Multistep Parameter Modulation

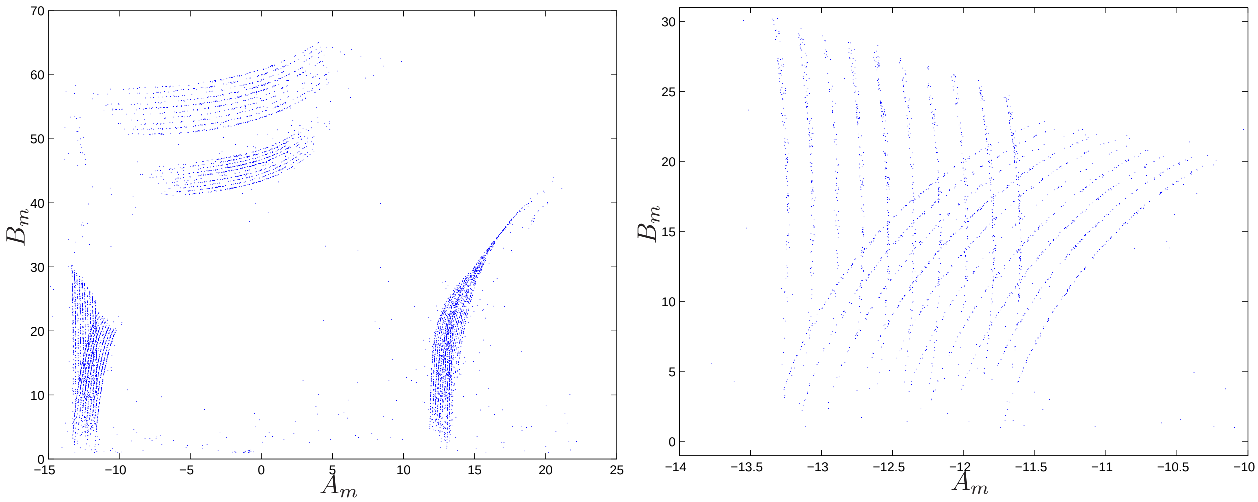
The security of multistep parameter modulation relies on the fact that the attacker has to assign 0-bits or 1-bits for all strips in the return map. Since there are $6n$ stripes in total, the success probability to make a right assignment is $\frac{1}{2^{6n}}$, i.e., the attack complexity is 2^{6n}. Note that the above analysis on the security is more rigorous, from the cryptographical point of view, than the one given in [Palaniyandi & Lakshmanan, 2001], where the latter enumerated the number of making wrong assignments under the assumption that the first assignment is correct. Of course, the order of the estimated attack complexity is the same.

[1]Different from x_s, there exist some small fluctuations in y_s. The faked maxima and minima induced by the small fluctuations should be removed from the return map; otherwise, the map will become completely meaningless. For the return map plotted in Fig. 3, therefore, if the difference between two consecutive maxima and minima is less than 1, they will be omitted.



a) a full view of the return map b) a local view of Segment 1

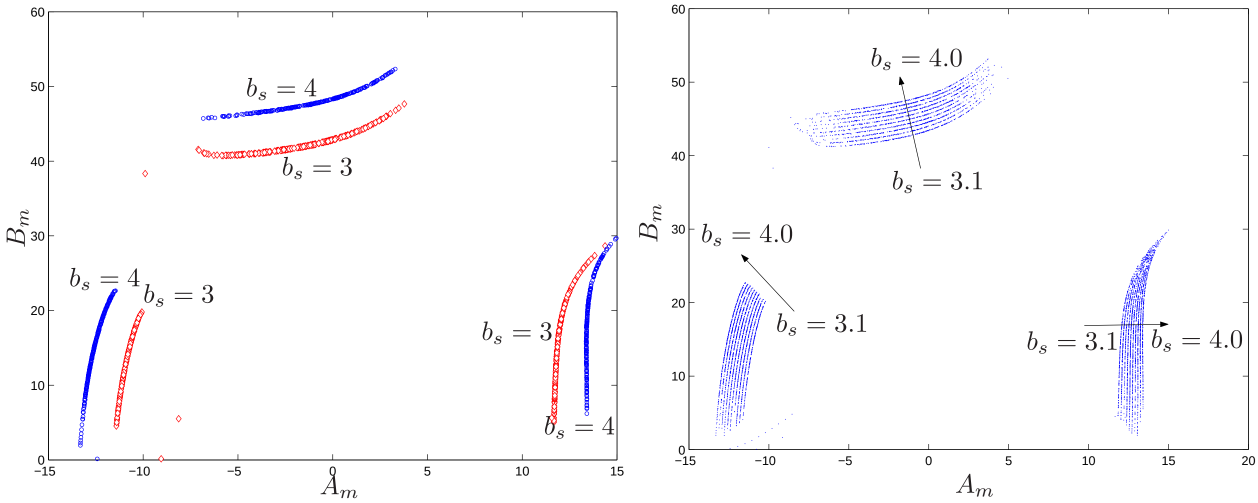
Figure 2: The return map constructed from x_s in multistep parameter modulation.



a) a full view of the return map b) a local view of Segment 1

Figure 3: The return map constructed in multistep parameter modulation with alternative x_s/y_s driving.

The above security estimation is based on the assumption that all $6n$ strips are independent of each other. However, we found that this assumption is not true and that there exists a deterministic relationship between the positions of the strips and the $2n$ different values of the modulated parameter b_s, and this relationship will dramatically reduce the attack complexity in all attacking scenarios. In Fig. 4a, the two return maps corresponding to $b_s = 3$ and $b_s = 4$ respectively are plotted to show such a deterministic relationship. One can see that the three segments corresponding to $b_s = 3$ are closer to the origin, while the three segments corresponding to $b_s = 4$ are farther. This means that there exist only two possibilities to assign the 0/1-bits to all strips in the chaotic switching scheme (see Fig. 1): for all three segments, assign 0-bit (or 1-bit) to the strip closer to the origin and 1-bit (or 0-bit) to the other one. If the relationship between b_0 and b_1 is also known to the attacker, he can uniquely determine the right assignment to completely break the plain-signal. Apparently, the above analysis can be easily generalized to multistep parameter modulation. Figure 4b shows the return maps corresponding to the 10 different values of b_s used in [Palaniyandi & Lakshmanan, 2001]. It can be seen that Fig. 4b is almost identical with the return map shown in Fig. 2a. Thus, it is easy to mark each strip of the return map shown in Fig. 2a with one of the $2n = 10$ possible values of b_s. For example, for Segment 1 shown in Fig. 2b, the i-th strip corresponds to $b_s = 3.0 + 0.1i$. This means that the task of assigning 0/1-bits to $6n$ strips is changed to another equivalent task of assigning 0/1-bits to $2n$ different values of b_s. Considering that there are n values corresponding to 0-bits and other n values to 1-bits, one can easily deduce that the number of all possible bit assignments is $2 \cdot \binom{2n}{n} = 2 \cdot \frac{(2n)!}{(n!)^2}$, which is $O\left(\frac{2^{2n}}{\sqrt{n}}\right)$ when $n \gg 1$ following Stirling's approximation [Weisstein, 2004b]. As a conclusion, the attack complexity is always much smaller than $O\left(2^{6n}\right)$, the original complexity estimated in [Palaniyandi & Lakshmanan, 2001]. Table 1 shows a comparison of the two complexities. From the cryptographical point of view, based on today's computer technology, a practically secure cryptosystem should have a complexity of order $O\left(2^{100}\right)$ [Schneier, 1996], which requires $n \geq 50$ following the data shown in Table 1. However, in this case, $4n \geq 200$ sub-systems have to be constructed to realize the decryption of the transmitted digital signal $m(t)$, which makes the implementation too costly for most practical applications. If the security can be relaxed to order of 2^{50}, $4n \geq 32$ sub-systems are enough to be practical in some applications (though still much more costly than other chaos-based secure communication systems). Note that the implementation cost will be acceptable in practice, if all the sub-systems can be realized with the same chaotic circuit.



a) the return maps corresponding to $b_s = 3$ and $b_s = 4$

b) the return maps corresponding to $b_s = 3.1, 3.2, \cdots, 3.9, 4.0$

Figure 4: A deterministic relationship between the return map and the modulated parameter b_s.

Table 1: A comparison of the real complexity $2 \cdot \binom{2n}{n}$ and the over-estimated complexity 2^{6n}.

n	8	10	12	14	16	18	20	25	30	35	40	45	50
$2 \cdot \binom{2n}{n} \approx$	$2^{14.7}$	$2^{18.5}$	$2^{22.4}$	$2^{26.3}$	$2^{30.2}$	$2^{34.1}$	2^{38}	$2^{47.8}$	$2^{57.7}$	$2^{67.6}$	$2^{77.5}$	$2^{87.4}$	$2^{97.3}$
2^{6n}	2^{48}	2^{60}	2^{72}	2^{84}	2^{96}	2^{108}	2^{120}	2^{150}	2^{180}	2^{210}	2^{240}	2^{270}	2^{300}

Note that one can extract some right bits even with a wrong bit assignment. For instance, for the example given in [Palaniyandi & Lakshmanan, 2001], 1-bits are assigned to $b_s \in \{3.1, 3.3, 3.5, 3.7, 3.9\}$ and 0-bits to $b_s \in \{3.2, 3.4, 3.6, 3.8, 4.0\}$, so one can get about 80% of right bits with the following bit assignment: 1-bits are assigned to $b_s \in \{3.1, 3.3, 3.5, 3.7, \mathbf{4.0}\}$, and 0-bits to $b_s \in \{3.2, 3.4, 3.6, 3.8, \mathbf{3.9}\}$, where the bold values correspond to wrong bits. Generally speaking, if there are $2i$ values corresponding to wrong bits, the bit error ratio (BER) at the attacker end will be i/n, i.e., the probability to get right bits is $1-(i/n)$. Note that when $i < n/2$, the attacker can simply flip all assigned bits to get a lower BER $(n-i)/n = 1-(i/n)$. From such a point of view, the worst bit assignment occurs when $i = \lfloor n/2 \rfloor$ or $\lceil n/2 \rceil$. Considering that the bit assignment can be regarded as an equivalent of the secret key, the above fact means that the decryption of multistep parameter modulation is insensitive to the mismatch of the secret key. However, such an insensitivity does not reduce the attack complexity by too much, since the number of wrong assignments corresponding to $i = \lfloor n/2 \rfloor$ or $\lceil n/2 \rceil$ is in the same order as the complexity $O\left(\frac{2^{2n}}{\sqrt{n}}\right)$ when $n \gg 1$: the number is $2 \cdot \binom{n}{\lfloor n/2 \rfloor} \cdot \binom{n}{n-\lfloor n/2 \rfloor} = 2 \cdot \binom{n}{\lfloor n/2 \rfloor} \cdot \binom{n}{\lceil n/2 \rceil} \approx O\left(\frac{2^{2n}}{n}\right)$, which is not much smaller than $O\left(\frac{2^{2n}}{\sqrt{n}}\right)$.

In cryptography, there are many different attacking scenarios [Schneier, 1996]. A cryptographically secure cryptosystem should be immune to all kinds of attacks. The above attack complexity of multistep parameter modulation is for the simplest attack – the ciphertext-only attack, where the attacker can only observe some ciphertexts. When some other attacking scenarios are available, the security of multistep parameter modulation will be dramatically downgraded.

Now, let us consider the security against known-plaintext and chosen-plaintext attacks, where the attacker can get or choose some plaintexts to carry out the attacks. Such attacks are feasible in some real applications and become more and more common in the digital networked world today. In known/chosen-plaintext attacks, it is obvious that the knowledge about some plaintexts means the knowledge about the bit assignment of the $6n$ strips: when $m(t) = 0$ (or 1), one immediately knows that the strip on which a point (A_m, B_m) lies corresponds to a 0-bit (or 1-bit), and then knows that other two strips marked with the same value of b_s also correspond to 0-bits (or 1-bits). That is, he can assign a 0-bit (or 1-bit) to the value of b_s corresponding to the distinguished strip. Once n 0-bits (or 1-bits) have been assigned to n different values of b_s, the attacker can directly assign 1-bits (or 0-bits) to all other undetermined values so as to complete the attack. For the number of required known/chosen plain-bits in the above attack, we have the following theoretical result.

Theorem 1 *Assume that b_s distributes uniformly over the set of $2n$ values and that any two values of b_s are independent of each other. Then, the average number of required known/chosen plain-bits in the above known/chosen-plaintext attack is $3n$.*

Proof: Denote the $k(\geq 1)$ known/chosen plain-bits by $B_1, \cdots, B_k \in \{0, 1\}$, and the corresponding values of b_s by $b_s^{(1)}, \cdots, b_s^{(k)}$. The condition that the attack is completed for the k known/chosen plain-bits equals to the following term: $n-1$ values corresponding to 0-bits (or 1-bits) have occurred in $b_s^{(1)}, \cdots, b_s^{(k-1)}$, and $b_s^{(k)}$ is the first occurrence of the last value. Considering that each value occurs with a uniform probability $p = \frac{1}{2n}$ and any two values are independent of each other, it is easy to get the probability that the attack stops with k known/chosen plain-bits, $P(k)$, as follows:

$$P(k) = \begin{cases} 0, & k < n \\ p(1-p)^{k-n}, & k \geq n. \end{cases} \tag{3}$$

Substituting $k' = k - n$ into the above equation, one can get $P(k') = p(1-p)^{k'}, \forall k' \geq 0$. It is obvious that $P(k')$ obeys a geometric distribution, and one can immediately deduce that $E(k') = p^{-1} = 2n$ [Weisstein, 2004a]. That is, $E(k) = E(k' + n) = E(k') + n = 3n$. The proof is thus completed. ■

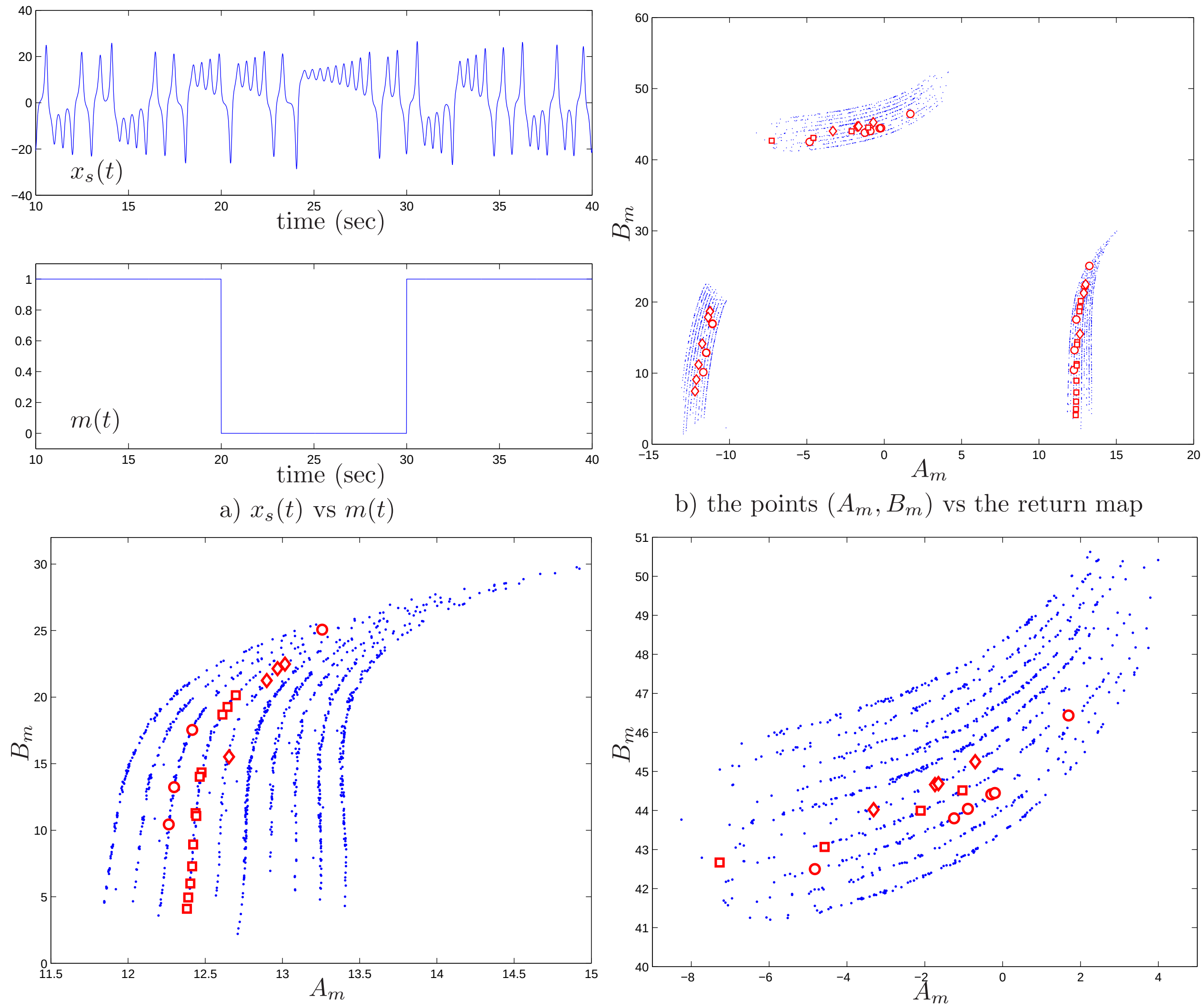
Since n cannot be too large to make the cryptosystem practical in real applications, the above theorem shows that multistep parameter modulation is not sufficiently secure against known/chosen-plaintext attacks. In Fig. 5, we give an example of known/chosen-plaintext attacks. It can be seen that three different values of b_s, i.e., nine strips in the return map, are successfully distinguished with only three known/chosen plain-bits.

4 Breaking Alternative Driving of Transmitter Variables

In this section, we consider how to break another countermeasure – alternative driving of transmitter variables. Following the example given in [Palaniyandi & Lakshmanan, 2001], we focus on the x/y-driving of the Lorenz system. Although the alternative driving can make the return map less clearer by introducing overlaps of the x_s-map and the y_s-map, it is found that the two overlapped sub-maps can be easily separated so that an attack can be carried out on the two sub-maps separately.

Since there are only two possible driving signals, the separation of the two driving signals can be simplified to the problem of detecting the times at which the driving signal, denoted by d_s here, changes from x_s to y_s or from y_s to x_s. This can be easily done by observing the differentiations of d_s, since the alternative driving will introduce breaking points at each switching time (i.e., discontinuities in d_s). Considering that chaotic signals $x_s(t)$ and $y_s(t)$ are both continuous, the switching times can be easily distinguished from sudden and large differentiations of d_s, where the word "sudden" means that the differentiation at a time t is much larger than the others around it. In Fig. 6, the first-order, second-order, 4th-order and 8th-order discrete-time differentiations of d_s are shown for demonstration, where the display range on the y-axis is always limited within $[-20, 20]$ to emphasize some sudden and large changes of differentiations with relatively small amplitudes. It can be seen that all switching times are sufficiently prominent in the 8th-order differentiations. Once the switching times are detected, one can easily separate the x_s-map and the y_s-map to break the multistep parameter modulation as discussed in the last section.

In fact, it is even possible to directly separate the two sub-maps without calculating differentiations of d_s. Observing Fig. 3, one can find that the overlaps of the two sub-maps are not very significant, which makes it possible to separate the two sub-maps directly from the alignment directions of consecutive points (A_m, B_m). When x_s-driving is used for odd bits and y_s for even bits, Fig. 7 shows the positions of the points (A_m, B_m) in the return map for $0 \leq t \leq 30$. In spite of the existence of a few error points and ambiguous points, which are mainly introduced by the faked maxima and minima near the switching times, it is still very easy to distinguish which driving signal was used from the alignment direction of the points (A_m, B_m) corresponding to the current bit (i.e., to the current value of b_s). The accidental errors and ambiguous points can be easily removed by filtering techniques.



a) $x_s(t)$ vs $m(t)$ b) the points (A_m, B_m) vs the return map

c) (A_m, B_m) vs the return map: Segment 2 d) (A_m, B_m) vs the return map: Segment 3

Figure 5: The known/chosen-plaintext attack to multistep parameter modulation, when $10 \leq t \leq 30$. Legend: $\diamond$ – $0 \leq t \leq 10$, $m(t) = 1$, $b_s = 3.5$; $\bigcirc$ – $10 \leq t \leq 20$, $m(t) = 1$, $b_s = 3.3$; $\square$ – $20 \leq t \leq 30$, $m(t) = 0$, $b_s = 3.4$.

Finally, we examine the attack complexity when both countermeasures are used in a secure communication system. Since there exist $12n$ strips, the average number of plain-bits in known/chosen-plaintexts attacks will be $2 \cdot 3n = 6n$, which means that the security against known/chosen-plaintext attacks is still rather weak. The security against ciphertext-only attacks is relatively higher: $\left(2 \cdot \binom{2n}{n}\right)^2$. However, note that an attacker can extract 50% of all plain-bits, even when he only exhaustively guesses the right bit assignment corresponding to the x_s-map or the y_s-map. Thus, strictly speaking, the security against ciphertext-only attacks is still in the order of $2 \cdot \binom{2n}{n}$, i.e., the same as the one under the condition that only the first countermeasure is used. As mentioned above, to make the designed secure communication system sufficiently secure, $n \geq 50$ is required.

5 Conclusion

To resist the return-map attack presented in [Pérez & Cerdeira, 1995], Palaniyandi & Lakshmanan [2001] proposed two countermeasures to enhance the security of the chaotic switching (i.e., binary parameter modulation) scheme. After refining the return-map attack by exploiting a deterministic relationship between the return map and the modulated parameter, this paper points out that these two countermeasures are not secure enough against known/chosen-plaintext attacks. Also, it is found that the security against ciphertext-only attacks cannot be ensured if the proposed secure communication system contains less than 200 sub-systems.

The cryptanalysis results given in this paper show that one has to use more powerful techniques to effectively resist return-map attacks. Recently, a new CSK scheme was proposed in [Xu & Chee, 2004] by introducing many false switching events. It is under study whether or not this new CSK scheme is secure against the

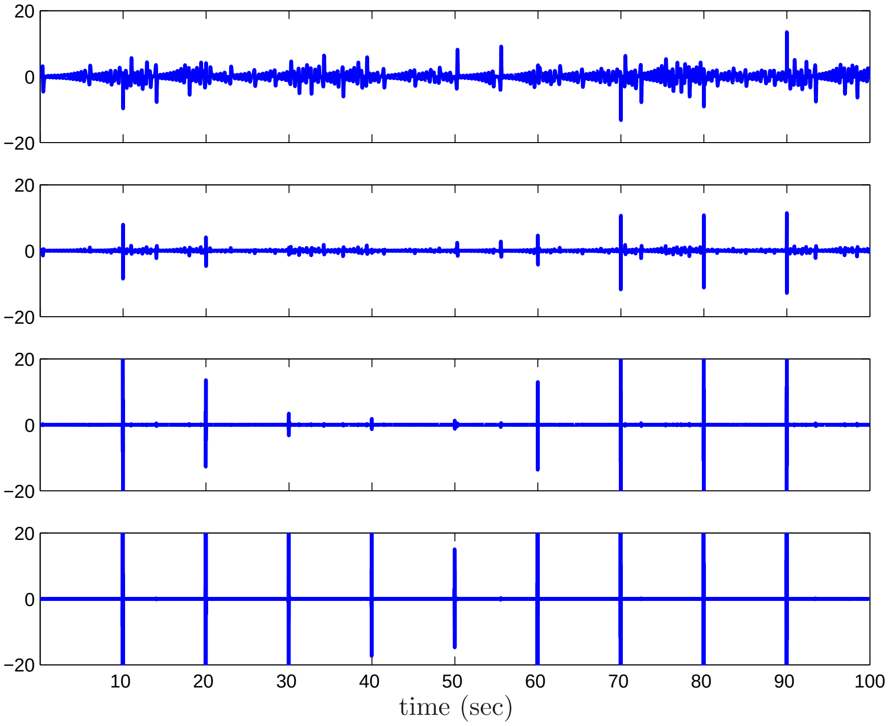


Figure 6: The first-order, second-order, 4th-order and 8-th order (from top to bottom) discrete-time differentiations of the transmitted signal d_s, where $\Delta t = 0.01$.

return-map attack described in this paper. At present, it is still an open problem to design a chaos-based secure communication system that is strong enough against all known attacks, and to find more powerful cryptanalysis tools to evaluate the security of various chaos-based cryptosystems.

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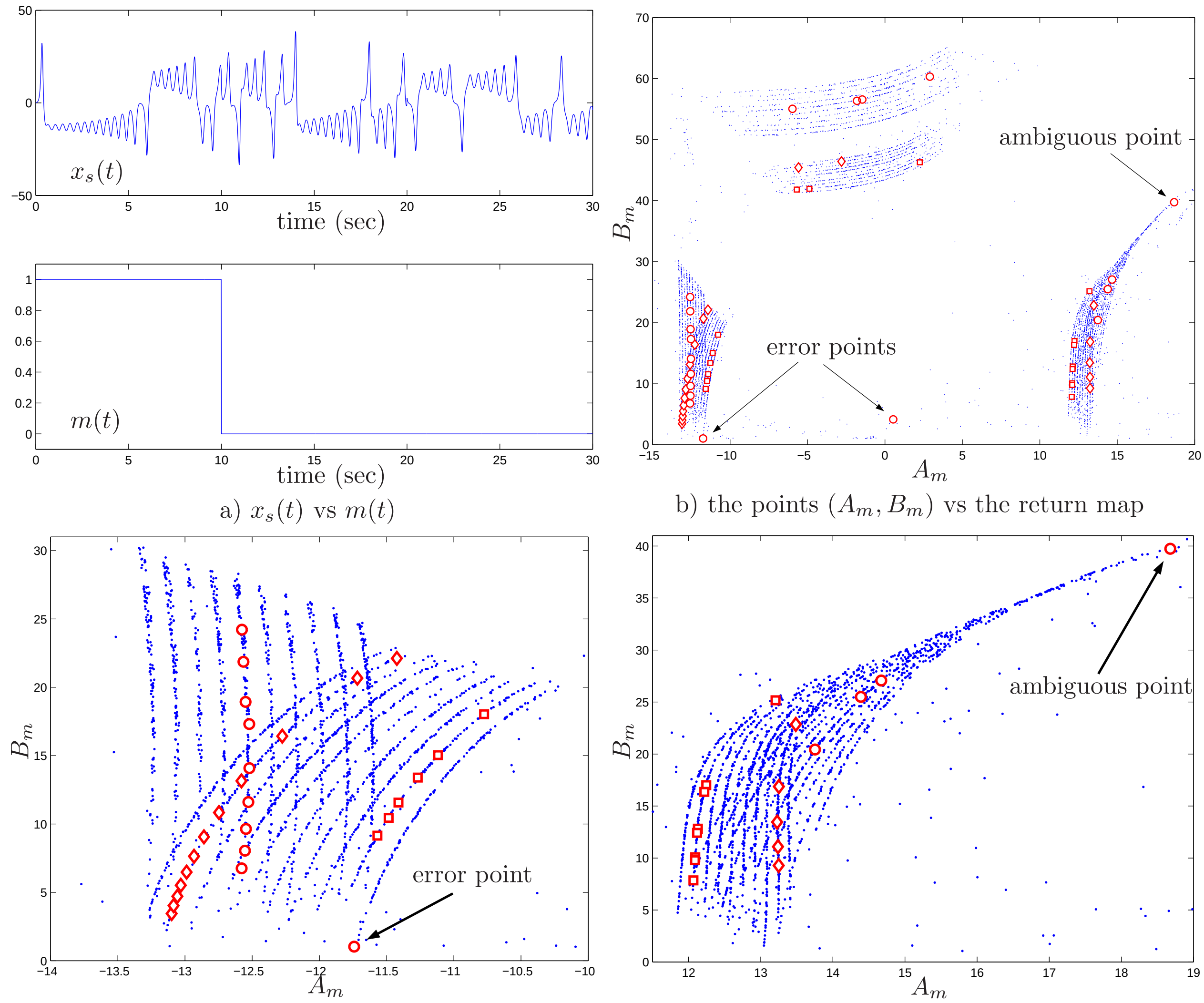

a) $x_s(t)$ vs $m(t)$ b) the points (A_m, B_m) vs the return map

c) (A_m, B_m) vs the return map: Segment 1 d) (A_m, B_m) vs the return map: Segment 2

Figure 7: The known/chosen-plaintext attack to multistep parameter modulation, when $10 \leq t \leq 30$. Legend: $\diamond$ – $0 \leq t \leq 10$, x_s-driving, $m(t) = 1$, $b_s = 3.9$; $\bigcirc$ – $10 \leq t \leq 20$, y_s-driving, $m(t) = 0$, $b_s = 3.6$; $\square$ – $20 \leq t \leq 30$, x_s-driving, $m(t) = 0$, $b_s = 3.2$.

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