Cryptanalysis of a chaotic block cipher with external key and its improved version

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Abstract

Recently, Pareek et al. proposed a symmetric key block cipher using multiple one-dimensional chaotic maps. This paper reports some new findings on the security problems of this kind of chaotic cipher: 1) a number of weak keys exists; 2) some important intermediate data of the cipher are not sufficiently random; 3) the whole secret key can be broken by a known-plaintext attack with only 120 consecutive known plain-bytes in one known plaintext. In addition, it is pointed out that an improved version of the chaotic cipher proposed by Wei et al. still suffers from all the same security defects.

Key words: chaos, encryption, known-plaintext attack, cryptanalysis, divide-and-conquer (DAC)
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1 Introduction

Due to some close and subtle relation between statistical properties of chaotic systems and cryptosystems, the idea of utilizing chaos to design digital ciphers and analog secure communication schemes has been attracting more and more attention during the past two decades [1–3].

Since 2003, Pareek et al. proposed three different cryptosystems based on one or more one-dimensional chaotic maps [4–6]. Unlike most existing chaotic ciphers [1], in the ciphers of Pareek et al., the initial conditions and/or the control parameter are not used as the secret keys, but derived from an external key instead, with the goal of obtaining a new way to achieve a higher level of security. The chaotic ciphers proposed in [4] and [5] have been cryptanalyzed by Alvarez et al. in [7], and by Wei et al. in [8], respectively. Wei et al. further proposed a remedy to improve the security of the original cipher against known-plaintext attacks.

This paper re-examines the security of the chaotic cipher designed in [5] and its improved version suggested in [8]. Three new security problems of the original cipher that were not reported in [8] are found: 1) there are a number of weak keys that cannot encrypt the plaintexts at all; 2) some important intermediate data of the cipher are not sufficiently random; 3) the secret key can be completely broken by a known plaintext attack with only 120 consecutive known plain-bytes in just one known-plaintext. In addition, it is found that the improved cipher developed in [8] still suffers from the same problems, thus failing to enhance the original cipher's security.

The rest of the paper is organized as follows. The next section gives a brief introduction to the original cipher of Pareek et al. and its improved version. Section 3 focuses on the above-mentioned security problems of the two chaotic ciphers under study. The last section concludes the paper.

2 The Cipher of Pareek et al. and its Improved Version

In the original cipher of Pareek et al. [5], the plaintext and the ciphertext are both arranged with 8-bit blocks, i.e., arranged byte by byte as follows: $P = P_1P_2 \cdots P_n$ and $C = C_1C_2 \cdots C_n$, where P_i, C_i are the i-th plain-byte and the i-th cipher-byte, respectively.

The secret key used in the cipher is a 128-bit integer K, represented as $K = K_1K_2 \cdots K_{16}$, where $K_i \in \{0, 1, \cdots, 255\}$ which is called the i-th sub-key in this paper[1]. The secret key is used to generate the initial conditions of four chaotic maps and the contents of two dynamic tables. Then, each plain-byte is masked by the output of one randomly-selected chaotic map after a number of iterations, under the control of the two dynamic tables. After a group of plain-bytes is encrypted, the two dynamic tables are updated following the current chaotic state of the selected chaotic map. The number of chaotic iterations and the group size are varying instead of being fixed. More precisely, the chaotic cipher works as follows.

(1) The following four chaotic maps are marked with map number $N = 0, 1, 2, 3$, respectively.

- $N = 0$ – logistic map: $f(x) = \lambda x(1 - x)$;
- $N = 1$ – tent map: $f(x) = \begin{cases} \lambda x, & \text{if } x < 0.5; \\ \lambda(1 - x), & \text{if } x \geq 0.5; \end{cases}$
- $N = 2$ – sine map: $f(x) = \lambda \sin(\pi x)$;
- $N = 3$ – cubic map: $f(x) = \lambda x(1 - x^2)$.

In [5], the control parameters of the above four chaotic maps are assigned as $\lambda = 3.99$, $\lambda = 1.97$, $\lambda = 0.99$ and $\lambda = 2.59$, respectively.

(2) The first dynamic table (DT1) stores the initial conditions (IC) of the four chaotic maps. Before the encryption process starts, the four initial conditions are all set to be the following value[2]:

$$\mathrm{IC} = \left(\sum_{i=1}^{16} \frac{K_i}{256} \right) \bmod 1 = \frac{\left(\sum_{i=1}^{16} K_i \right) \bmod 256}{256}. \tag{1}$$

(3) Each entry of the second dynamic table (DT2) stores three distinct values: the selected chaotic map that encrypts a group of plain-bytes, the number of plain-bytes in a group that is encrypted by the corresponding chaotic map, and the number of iterations of the corresponding chaotic map for encrypting each plain-byte, which are denoted by N, B and IT, respectively. Given a linear congruential pseudorandom number generator (LCG),

$$Y_0 = \lfloor 100 \times \mathrm{IC} \rfloor, \tag{2}$$
$$Y_n = (5Y_{n-1} + 1) \bmod 16, \text{ when } n \geq 1, \tag{3}$$

the three values of the n-th entry in DT2 are determined as follows[3]:

[1] In [5], K_i is called "session key". However, such a term may cause some confusion, since "session keys" are generally used to denote randomly-generated keys in cryptographical protocols.

[2] Note that we use an equivalent formula to replace Eqs. (4) and (5) in [5], trying to give a clearer representation. Here "mod 1" means subtracting the integer part and keeping only the fractional part, which lies in the half-open interval $[0, 1)$.

[3] Note that Y_0 is not confined in $\{0, \cdots, 15\}$, so it is just used as the seed of the LCG and should not be considered as part of the LCG sequence to generate the

$$N_n = Y_n \bmod 4, \tag{4}$$

$$B_n = Y_n, \tag{5}$$

$$\mathrm{IT}_n = K_{Y_n+1}. \tag{6}$$

In [5], it's said that DT2 has a number of rows equal to the total number of session keys, which means that the number of entries in DT2 is 16.

(4) The encryption process runs by reading each entry of DT2. For the n-th entry, the chaotic map marked with number N_n is chosen to encrypt a group of B_n plain-bytes. Each plain-byte P_i is masked by the chaotic state after IT_n iterations of the chosen chaotic maps, according to the following rule:

$$C_i = \left(P_i + \lfloor X_{\mathrm{new}} \cdot 10^5 \rfloor\right) \bmod 256, \tag{7}$$

where X_{new} is the new chaotic state after the IT_n iterations. After each plain-byte is encrypted, IC of the chosen chaotic map in DT1 is updated as X_{new}. Once DT2 is exhausted, substitute the latest value of IC in DT1 into Eq. (2) to reset Y_0, and then repeat Eqs. (3) to (6) for 16 times to update all entries of DT2 for future encryption.

(5) The decryption procedure is similar to the above encryption procedure, by replacing Eq. (7) with the following one:

$$P_i = \left(C_i - \lfloor X_{new} \cdot 10^5 \rfloor\right) \bmod 256. \tag{8}$$

Wei et al. in [8] pointed out that the above cipher works like a stream cipher, so a key-stream $\{(C_i - P_i) \bmod 256\}$ can be constructed in known-plaintext attacks and then be used as an equivalent of the secret key K to decrypt other ciphertexts. To overcome this security problem, Wei et al. suggested a remedy to modify Eq. (4), as follows:

$$N_n = (Y_n \bmod 4) \oplus \left(\bigoplus_{i=0}^{n-1} P_i \bmod 4\right), \tag{9}$$

where P_i is the i-th plain-byte, $\oplus$ denotes the bitwise XOR operation, and $P_0 = 0$.

3 Cryptanalysis

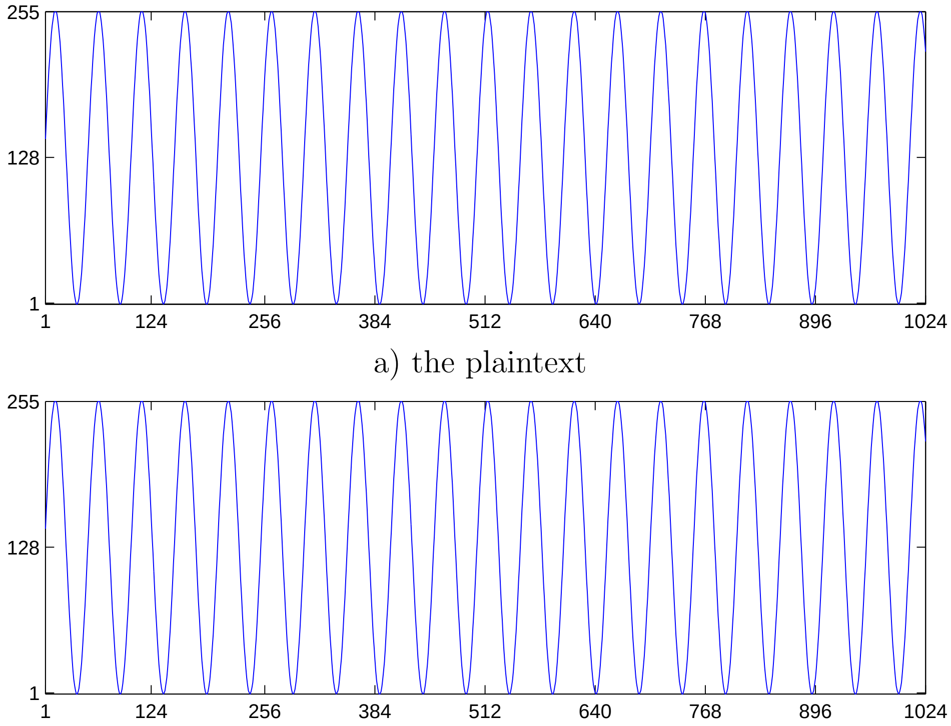
In addition to the defect of the original cipher of Pareek et al. [5] pointed out in [8], we found some other security problems that exist in both the original cipher and the improved version proposed in [8].

entries of DT2.

3.1 Weak Keys

Observing Eq. (1), one can see the number of all possible values of IC is only $256 = 2^8$, namely, $\frac{0}{256} \sim \frac{255}{256}$. Since $x = 0$ is a fixed point common to all the four chaotic maps, IC $= \frac{0}{256}$ will cause all chaotic states to be zero, which means that $C_i = P_i$, $\forall i$. In this case, the chaotic cipher does not work at all and the corresponding key is an extremely weak key. To make IC $= \frac{0}{256}$, one has $\sum_{i=1}^{16} K_i \equiv 0 \pmod{256}$. Then, one can calculate the number of such weak keys to be $2^{16\times 8}/256 = 2^{15\times 8} = 2^{120}$. Figure 1 shows the encryption result when a weak key $K = 61624D51595F888A434487885C5E483D$ (represented in hexadecimal format) is used to encrypt a sinusoidal waveform.

Additionally, to ensure a higher level of security, the value of IT should not be too small, which means that each sub-key should not be too short. This will further reduce the key space.



a) the plaintext

b) the ciphertext

Fig. 1. The encryption result of a sinusoidal waveform with one weak key, “61624D51595F888A434487885C5E483D”.

Finally, it is worth mentioning that the same kind of weak keys also exists in the chaotic cipher proposed by Pareek et al. in [4], due to the similarity between the two ciphers. This weakness had not been pointed out in Alvarez et al.’s cryptanalysis paper [7].

3.2 Weak Randomness of DT2

The second dynamic table DT2 is generated in a pseudorandom way using a LCG and controlled by the secret key. Such generators are easy to implement and pass many statistical tests, thus leading to believe that they are good candidates for generating strong pseudorandom sequences for cryptographical applications. However, these sequences are predictable: given a piece of the sequence, it is possible to reconstruct all the rest even if the parameters are unknown [9]. Therefore, the use of linear congruential generators in cryptography is totally discouraged. Furthermore, the choice of parameters for the LCG in [5] is most unfortunate. Using a prime number as the modulus of the LCG would have yielded better results, but by using 16 as modulus, the randomness of its sequences is null. In fact, the sequence is a unique cycle where the start value is the seed of the LCG. The known-plaintext attack discussed in the next subsection benefits from the lack of randomness of DT2, which reduces the attacking complexity.

In the following, we prove some mathematical results on the LCG sequence $\{Y_n\}$ and the map-number sequence $\{N_n\}$. It can be seen that the two sequences are far from having "good" randomness.

Lemma 1 *Given an initial integer Y_0 and a sequence $\{Y_n\}_{n\geq 1}$, where $Y_n = (5Y_{n-1}+1) \bmod 16$. We have $Y_n = \left(5^n Y_0 + \sum_{i=n-1}^{0} 5^i\right) \bmod 16$.*

Proof: We prove this lemma via mathematical induction.

When $n = 1$, $Y_1 = (5Y_0 + 1) \bmod 16$, so the lemma is true.

Assuming $Y_n = \left(5^n Y_0 + \sum_{i=n-1}^{0} 5^i\right) \bmod 16$ holds for $1 \leq n \leq k$, we prove the lemma for the case of $n = k+1 \geq 2$. From $Y_n = (5Y_{n-1}+1) \bmod 16$, we have

$$\begin{aligned} Y_{k+1} &= (5Y_k + 1) \bmod 16 \\ &= \left(5\left(\left(5^k Y_0 + \sum_{i=k-1}^{0} 5^i\right) \bmod 16\right) + 1\right) \bmod 16 \\ &= \left(5\left(5^k Y_0 + \sum_{i=k-1}^{0} 5^i\right) + 1\right) \bmod 16 \\ &= \left(5^{k+1} Y_0 + \sum_{i=k}^{0} 5^i\right) \bmod 16. \end{aligned}$$

Thus, the lemma is proved. ■

Theorem 1 *Given an initial value Y_0 and a sequence $\{Y_n\}_{n\geq 1}$, where $Y_n =$*

$(5Y_{n-1}+1) \bmod 16$. *We have* $Y_n = (2n^2 + (4Y_0 - 1)n + Y_0) \bmod 16$.

Proof: From Lemma 1, we have

$$\begin{aligned}
Y_n &= \left(5^n Y_0 + \sum_{i=n-1}^{0} 5^i\right) \bmod 16 = \left(5^n Y_0 + \frac{5^n - 1}{5-1}\right) \bmod 16 \\
&= \left((1+4)^n Y_0 + \frac{(1+4)^n - 1}{4}\right) \bmod 16 \\
&= \left(\sum_{i=0}^{n} \binom{n}{i} 4^i Y_0 + \frac{\sum_{i=0}^{n} \binom{n}{i} 4^i - 1}{4}\right) \bmod 16 \\
&= \left((1+4n)Y_0 + \left(n + \binom{n}{2} 4\right)\right) \bmod 16 \\
&= \left(2n^2 + (4Y_0 - 1)n + Y_0\right) \bmod 16.
\end{aligned}$$

This completes the proof of the theorem. ■

Corollary 1 *Given an initial integer* Y_0 *and a sequence* $\{Y_n\}_{n\geq 1}$, *where* $Y_n = (5Y_{n-1}+1) \bmod 16$. *The sequence has a period of* 16.

Proof: Assume the period of the sequence $\{Y_n\}$ is T. From Theorem 1, we can get $Y_{n+16} - Y_n \equiv 0 \pmod{16}$. This means that $T|16$, i.e., $T \in \{1, 2, 4, 8, 16\}$. Again, from Theorem 1, we have

$$\begin{aligned}
Y_{n+8} - Y_n \equiv &\left(2(n+8)^2 + (4Y_0 - 1)(n+8) + Y_0\right) \\
&- \left(2n^2 + (4Y_0 - 1)n + Y_0\right) \pmod{16} \\
\equiv &\, 8 \pmod{16}.
\end{aligned}$$

Since $Y_n, Y_{n+8} \in \{0, \cdots, 15\}$, the above result means $Y_{n+8} \neq Y_8$. That is, $T > 8 \Rightarrow T = 16$, which proves the corollary. ■

Remark 1 *From Theorem 1, it is obvious that there are only 16 distinct sequences of* $\{Y_n\}_{n\geq 1}$, *shown as follows* ($Y_1 \sim Y_{16}$):

1	6	15	12	13	2	11	8	9	14	7	4	5	10	3	0	···
6	15	12	13	2	11	8	9	14	7	4	5	10	3	0	1	···
15	12	13	2	11	8	9	14	7	4	5	10	3	0	1	6	···
12	13	2	11	8	9	14	7	4	5	10	3	0	1	6	15	···
13	2	11	8	9	14	7	4	5	10	3	0	1	6	15	12	···
2	11	8	9	14	7	4	5	10	3	0	1	6	15	12	13	···
11	8	9	14	7	4	5	10	3	0	1	6	15	12	13	2	···
8	9	14	7	4	5	10	3	0	1	6	15	12	13	2	11	···
9	14	7	4	5	10	3	0	1	6	15	12	13	2	11	8	···
14	7	4	5	10	3	0	1	6	15	12	13	2	11	8	9	···
7	4	5	10	3	0	1	6	15	12	13	2	11	8	9	14	···
4	5	10	3	0	1	6	15	12	13	2	11	8	9	14	7	···
5	10	3	0	1	6	15	12	13	2	11	8	9	14	7	4	···
10	3	0	1	6	15	12	13	2	11	8	9	14	7	4	5	···
3	0	1	6	15	12	13	2	11	8	9	14	7	4	5	10	···
0	1	6	15	12	13	2	11	8	9	14	7	4	5	10	3	···

It can be seen that the 16 sequences actually represent the same sequence with different starting points. This is a common feature of discrete maps defined over a finite field and with a maximal period [10].

Corollary 2 *Given an initial integer* Y_0 *and a sequence* $\{Y_n\}_{n\geq 1}$*, where* $Y_n = (5Y_{n-1}+1) \bmod 16$*. Then,* $\forall n \geq 1$*,* $\{Y_n, Y_{n+4}, Y_{n+8}, Y_{n+12}\}$ *must be one of the following four sets:* $\{0, 12, 8, 4\}$*,* $\{1, 13, 9, 5\}$*,* $\{2, 14, 10, 6\}$ *and* $\{3, 15, 11, 7\}$*.*

Proof: From Theorem 1, $Y_{n+4} - Y_n \equiv (2(n+4)^2 + (4Y_0 - 1)(n+4) + Y_0) - (2n^2 + (4Y_0 - 1)n + Y_0) \equiv (4Y_0 - 1)4 \equiv -4 \pmod{16}$. Since $Y_n \in \{0, 1, \cdots, 15\}$, the corollary is immediately proved. (The corollary can also be proved by exhaustively examining all 16 distinct sequences of $\{Y_n\}$.) ■

Theorem 2 *Given a sequence* $\{Y_n\}_{n\geq 1}$*, where* $Y_n = (5Y_{n-1}+1) \bmod 16$ *for* $n \geq 2$*. Then, assuming* $N_n = Y_n \bmod 4$*, we have* $N_n = (n + Y_0) \bmod 4$*.*

Proof: Substituting the result of Theorem 1 into $N_n = Y_n \bmod 4$, we have $N_n = Y_n \bmod 4 = (2n^2 + (4Y_0 - 1)n + Y_0) \bmod 4 = (2n^2 - n + Y_0) \bmod 4$. Note that $(2n^2 - n) - n \equiv 2n(n-1) \equiv 0 \pmod 4$, so $2n^2 - n \equiv n \pmod 4$. This immediately leads to $N_n = (n + Y_0) \bmod 4$ and proves the theorem. ■

Corollary 3 *Given an initial integer Y_0 and two sequences $\{Y_n\}_{n\geq 1}$, $\{N_n\}_{n\geq 1}$, where $Y_n = (5Y_{n-1} + 1) \bmod 16$ and $N_n = Y_n \bmod 4$. Then, the sequence $\{N_n\}_{n\geq 1}$ has a periodicity of 4, and must be one of the following four sequences: $\{1, 2, 3, 0, \cdots\}$, $\{2, 3, 0, 1, \cdots\}$, $\{3, 0, 1, 2, \cdots\}$ and $\{0, 1, 2, 3, \cdots\}$.*

Proof: This corollary is a straightforward consequence of Theorem 2. ■

3.3 Breaking the Secret Key by a Known-Plaintext Attack

In [8, Sec. 4], Wei et al. pointed out that the original cipher of Pareek et al. is vulnerable to known-plaintext attacks. However, Wei et al.'s attack does not break the secret key itself, but only reveals an equivalent of the secret key – the key stream $\{(C_i - P_i) \bmod 256 = \lfloor X_{\text{new},i} \cdot 10^5 \rfloor \bmod 256\}$. The main disadvantage of this attack is that it can only completely break a ciphertext as long as the whole keystream involved is recovered. In the real world, this means than all plain-bytes beyond the maximal length of known plaintexts cannot be recovered.

In this subsection, we report a practical known-plaintext attack to completely reveal the secret key, with only 120 consecutive known plain-bytes in just one known plaintext, with rather small computational complexity. This attack is very practical in real world scenarios.

From Corollary 3, one can see that $\forall n \in \{1, 2, 3, 4\}$, the plain-bytes in the n, $(n+4)$, $(n+8)$, $(n+12)$-th groups are encrypted by the chaotic map numbered with $N_n = N_{n+4} = N_{n+8} = N_{n+12}$. At the same time, from Corollary 1, the 16 ITs in DT2 form a permutation of the 16 sub-keys $K_1, \cdots, K_{16}$. The two facts mean that we can try to **separately** break the sub-keys used for each chaotic map. If such a divide-and-conquer (DAC) attack really works, the total complexity of revealing all 16 sub-keys will be dramatically reduced as compared with exhaustively searching them throughout the whole key space.

It is found that a three-stage DAC attack shown below works well following the above idea.

- *Stage 1 – exhaustively guessing* IC *in Eq. (1) and 4 sub-keys (i.e.,* IT*s) used by one chaotic map numbered with N_n.*

  For each guessed value of IC, the chaotic map is chosen to ensure that $\{B_n, B_{n+4}, B_{n+8}, B_{n+12}\}$ does not contain zero[4]. To eliminate incorrectly guessed values of IC, the repeated use of IT_n in each group is employed –

[4] Corollary 3 ensures that there are always three chaotic maps of this kind. We can randomly choose one from the three.

all B_n chaotic states in the n-th group should correspond to the same value of $\lfloor X_{\text{new}} \times 10^5 \rfloor \bmod 256 = (C_i - P_i) \bmod 256$.

The output of this stage will be some candidate values of IC, each of which corresponds to 4 revealed sub-keys. Without loss of generality, assume that the chaotic map has a uniform invariant distribution. Then, we can calculate the probability of getting a wrong candidate value

$$P_e = \frac{256^4}{256^{B_n + B_{n+4} + B_{n+8} + B_{n+12}}}.$$

It follows from Corollary 3 that

$$P_e \leq \frac{256^4}{256^{1+13+9+5}} = 256^{-24} = 2^{-192}.$$

To further minimize the value of P_e, for each guessed value of IC, one can chose the map corresponding to $\{B_n, B_{n+4}, B_{n+8}, B_{n+12}\} = \{3, 15, 11, 7\}$. In this way, P_e will be minimized to be $256^4/256^{3+15+11+7} = 256^{-32} = 2^{-256}$. Thus, it is an extremely rare event to get more than one candidate value of IC in practice[5].

- *Stage 2 – exhaustively searching other 11 sub-keys (i.e.,* IT*s) used by other three chaotic maps.*

  Once the value of IC is determined, we can use a similar method in Stage 1 to determine the sub-keys used by other three chaotic maps. Note that the sub-key corresponding to $B_n = 0$ cannot be found, since no any plain-byte is encrypted with this sub-key. So, only 11 sub-keys can be revealed in this stage and the last one is left for the next stage.

- *Stage 3 – revealing the last unknown sub-key via Eq. (1).*

  In the above two stages, one can successfully get the value of IC and break 15 sub-keys. The last sub-key can be determined via Eq. (1). Assuming the undetermined sub-key is K_j, we have

$$K_j = \left(256 \times \text{IC} - \sum_{\substack{1 \leq i \leq 16 \\ i \neq j}} K_i \right) \bmod 256. \tag{10}$$

Now, let us estimate the computational complexity of this attack. First, the computational complexity of Stage 3 is very small, so we can consider only the first two stages. By enumerating the number of guessed values of IC and the number of all chaotic iterations, we can deduce that the computational complexity of Stage 1 is not greater than $O(255 \times 256 \times (3 + 15 + 11 + 7)) \approx O(2^{21})$ and Stage 2 is not greater than $O(256 \times (3 + 15 + 11 + 7 + 2 + 14 + 10 + 6 + 0 + 12 + 8 + 4)) \approx O(2^{14.5})$. As a whole, the total complexity of

[5] Even when such a rare event happens, one can verify all the candidate values by choosing another chaotic map. This will further eliminate wrong candidate values and eventually leave only the correct one.

the DAC attack is mainly determined by Stage 1, which is not greater than $O(2^{21})$. Attacks with such small complexity can be easily carried out on a PC.

Besides the very small computational complexity, the required number of known plain-bytes in an attack is also very small – only $\sum_{i=1}^{16} B_i = \sum_{i=0}^{15} i = 120$ plain-bytes in one known plaintext are enough.

The above analysis shows that the proposed DAC attack is very efficient. To further validate the feasibility of the attack, a real attack was carried out with one known plaintext as shown in Fig. 1a) and the corresponding ciphertext shown in Fig. 2. The breaking results obtained in all the three stages are given in Table 1. With the broken sub-keys, one can immediately get the whole secret key, $K = K_1 \cdots K_{16} = BCDA178E512131422E859F086E2E884F$ (represented in hexadecimal format).

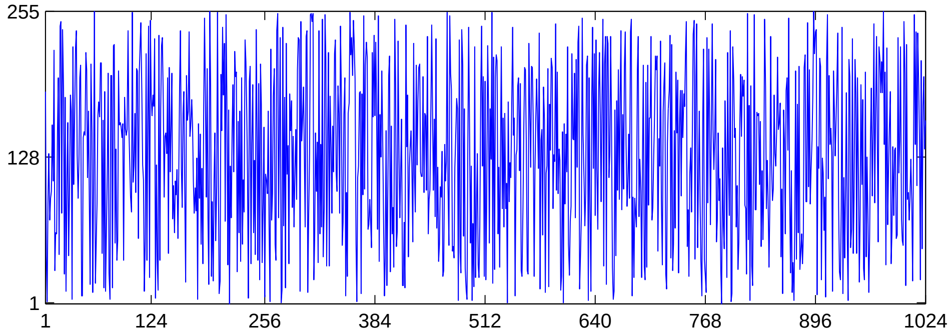


Fig. 2. The ciphertext of the sinusoidal waveform shown in Fig. 1a), with $K = BCDA178E512131422E859F086E2E884F$.

3.4 Security Problem of Wei et al.'s Version

The improved version of the original cipher, proposed by Wei et al. in [8], employs plaintext feedback to enhance the security against the simple keystream-based known-plaintext attack. However, even this cipher cannot resist the DAC attack proposed-above in this paper, because this attack does not depend on the relation between the keystream and the plaintext. Of course, in the cipher of Wei et al., because the periodicity of $\{N_n\}_{n\geq 1}$ is destroyed by the plaintext feedback, the performance of the DAC attack may be complicated slightly. The main influence includes the following two aspects.

First, in Stage 1, the plaintext feedback influences the manner of choosing the target chaotic map, since now the n-th chaotic map generally does not correspond to $\{B_n, B_{n+4}, B_{n+8}, B_{n+12}\}$, but to a set $\{B_{n_1}, B_{n_2}, \cdots, B_{n_i}\}$ whose size depends on the plaintext. To minimize the value of P_e, we should choose the target chaotic map as the one with the maximal value of $\sum_{j=1}^{i} B_{n_j}$. Since $\sum_{j=1}^{16} B_j = \sum_{j=1}^{16}(j-1) = 120$, we can deduce $\sum_{j=1}^{i} B_{n_j} \geq 120/4 = 30$. This

Table 1
The stage-by-stage breaking results of a real example of the proposed known-plaintext attack.

	Stage 1	Stage 2	Stage 3
IC	$\frac{237}{256}$		
$K_{14} = \mathrm{IT}_1$	146		
$K_3 = \mathrm{IT}_2$		23	
$K_{12} = \mathrm{IT}_3$		8	
$K_9 = \mathrm{IT}_4$		46	
$K_{10} = \mathrm{IT}_5$	133		
$K_{15} = \mathrm{IT}_6$		136	
$K_8 = \mathrm{IT}_7$		66	
$K_5 = \mathrm{IT}_8$		81	
$K_6 = \mathrm{IT}_9$	33		
$K_{11} = \mathrm{IT}_{10}$		159	
$K_4 = \mathrm{IT}_{11}$		142	
$K_1 = \mathrm{IT}_{12}$			188
$K_2 = \mathrm{IT}_{13}$	218		
$K_7 = \mathrm{IT}_{14}$		49	
$K_{16} = \mathrm{IT}_{15}$		79	
$K_{13} = \mathrm{IT}_{16}$		110	

means that $P_e \leq \frac{256^4}{256^{30}} = 256^{-26} = 2^{-208}$. So, it is still an extremely rare event to get more than one candidate value after Stage 1 is completed.

Second, in Stage 2, for one or two chaotic maps, the value of $\sum_{j=1}^{i} B_{n_j}$ may not be large enough to uniquely determine the values of some sub-keys. In this case, only 120 plain-bytes will not be enough to recover all sub-keys. Nevertheless, the probability of this event is not too large[6], so these undetermined sub-keys will be gradually broken with the accumulation of more known plain-bytes.

Finally, the following two points on the security of Wei et al.'s improved cipher are worth mentioning: 1) in the chosen-plaintext counterpart of the DAC attack, the plaintext feedback mechanism can be completely circumvented by

[6] It is not easy to theoretically deduce this probability. Assuming all chaotic maps satisfy $P_e \leq 10^{-4}$, we found the probability is not greater than 0.06 with 300,000 random experiments in Matlab.

choosing all plain-bytes to be zero; 2) the plaintext feedback cannot rule out the existence of weak keys and the weak randomness of $\{B_n\}_{n\geq 1}$. To sum up, Wei et al.'s remedy is not essentially improving the security of the original cipher of Pareek et al.

4 Conclusions

In this paper, the security of a recently-proposed cipher based on multiple one-dimensional chaotic maps [5] has been re-examined, showing that a previous cryptanalysis [8] did not reveal many major security problems. As a result, a number of weak keys and weak pseudorandomness of some intermediate data were discovered and distinguished, and an efficient known-plaintext attack can be recommended to completely reveal the whole secret key. The proposed attack has a very small computational complexity, which works with only 120 plain-bytes in one known plaintext. In addition, it is found that an improved version of the original cipher, proposed in [8], also suffers from the same security problems. The cryptanalysis given in this paper thus discourages the use of the chaotic cipher proposed in [5, 8], especially when known-plaintext attacks are possible.

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