Breaking a chaos-based secure communication scheme designed by an improved modulation method

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Abstract

Recently Bu and Wang [1] proposed a simple modulation method aiming to improve the security of chaos-based secure communications against return-map-based attacks. Soon this modulation method was independently cryptanalyzed by Chee et al. [2], Wu et al. [3], and Álvarez et al. [4] via different attacks. As an enhancement to the Bu-Wang method, an improving scheme was suggested by Wu et al. [3] by removing the relationship between the modulating function and the zero-points. The present paper points out that the improved scheme proposed in [3] is still insecure against a new attack. Compared with the existing attacks, the proposed attack is more powerful and can also break the original Bu-Wang scheme. Furthermore, it is pointed out that the security of the scheme proposed in [3] is not so satisfactory from a pure cryptographical point of view. The synchronization performance of this class of modulation-based schemes is also discussed.

1 Introduction

Since the early 1990s, the use of chaotic systems in cryptography has been extensively investigated. There are two main classes of chaotic cryptosystems: analog [5, 6] and digital [7]. Most analog chaotic cryptosystems are secure

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communication systems based on synchronization of the sender and the receiver chaotic systems [8], where a signal is transmitted over a public channel from the sender to the receiver, and decryption of the plain-signal is realized via chaos synchronization at the receiver end. According to the encryption structures, most analog chaos-based secure communication systems can be classified into four categories: chaotic masking [9, 10], chaotic switching or chaotic shift keying (CSK) [11, 12], chaotic modulation [13, 14], and chaotic inverse system [15]. Meanwhile, some cryptanalysis work has also been developed, and many different attacks have been proposed to break different types of chaos-based secure communication systems: return-map-based attacks [16–18], spectral analysis attacks [19, 20], generalized-synchronization-based attacks [21, 22], short-time-period-based attacks [23, 24], prediction-based attacks [25, 26], parameter-identification-based attacks [27–30], and so on.

Since many early chaos-based secure communication systems are found insecure against various attacks, how to design robust chaos-based secure communication systems against the existing attacks is always a real challenge. The following three types of countermeasures have been proposed in the literature: 1) using more complex dynamical systems, such as hyperchaotic systems or multiple cascaded heterogeneous chaotic systems [31]; 2) introducing traditional ciphers into chaotic cryptosystems [32]; 3) introducing an impulsive (also named sporadic) driving signal instead of a continuous signal to realize modulation and synchronization [33]. The first countermeasure was found lately remaining insecure against some specific attacks [26,28,34,35], and some security defects of the second have also been reported [36].

In [1], a simple modulation method was proposed by Bu and Wang to enhance the security of some chaos-based secure communications against return-map-based attacks. A periodic signal is used to modulate the transmitted signal in order to effectively blur the reconstructed return map. Although this modulation method can frustrate return-map-based attacks, it was soon broken independently in [2–4] with other different attacks, some of which rely on the embedded periodicity of zero-crossing points of the modulating signal. To further improve the security of the original Bu-Wang modulation method, a modified modulating signal was then suggested in [3] to cancel the embedding regular occurrence of zero-crossing points in the transmitted signal.

This paper points out that the modified modulation method proposed in [3] is still insecure against a new attack that cab restore the return map. The secret parameters of the modulating signal can be approximately identified via the proposed attack. Compared with other attacks, the new attack is more powerful which can also break the original Bu-Wang modulation scheme. Furthermore, it will be pointed out that from a cryptographical point of view the modulation method itself is not so satisfactory in enhancing the security of the chaotic cryptosystems even when the modulating signal is secure enough.

The rest of this paper is organized as follows. In Sec. 2, the modulation method and related attacks are briefly introduced. Section 3 shows how the new attack breaks the scheme based on the modified modulation method. Some general discussions on practical performance of this class of modulation-based methods under study are given in Sec. 4. The last section concludes the paper.

2 The modulation method and related attacks

To introduce the modulation-based method, the Lorenz system is used to construct a secure communication system. The sender system is

$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1), \\ \dot{x}_2 &= rx_1 - x_2 - x_1x_3, \\ \dot{x}_3 &= x_1x_2 - bx_3 \end{aligned} \tag{1}$$

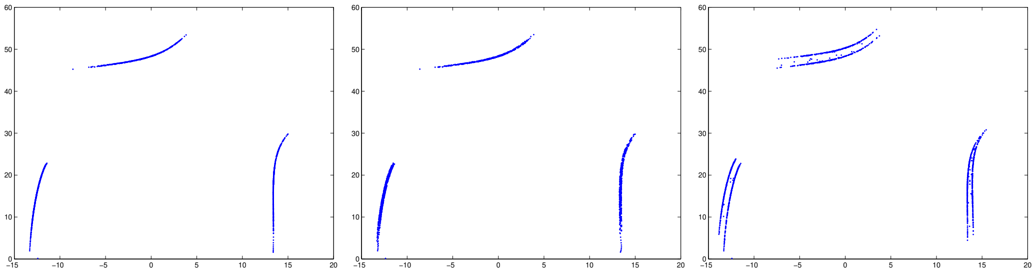
and the receiver system is

$$\begin{aligned} \dot{y}_1 &= \sigma(y_2 - y_1) + c(s(\mathbf{x}, t) - s(\mathbf{y}, t)), \\ \dot{y}_2 &= ry_1 - y_2 - y_1y_3, \\ \dot{y}_3 &= y_1y_2 - by_3, \end{aligned} \tag{2}$$

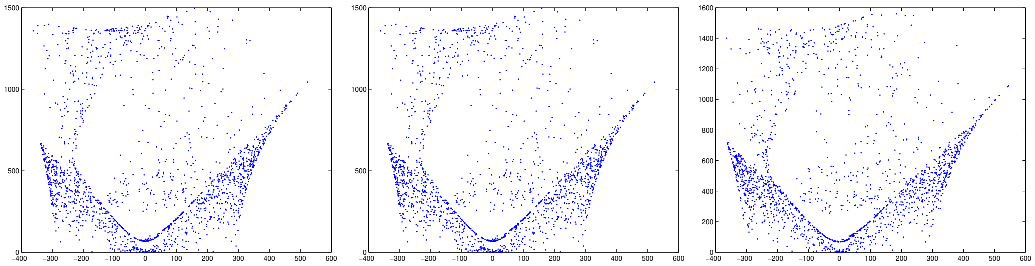
where σ, b, r are parameters of the system, $\mathbf{x} = (x_1, x_2, x_3)$ and $\mathbf{y} = (y_1, y_2, y_3)$ are state variables, c denotes the coupling strength, and $s(\mathbf{x}, t) = g(t)x_1(t)$ is the ciphertext transmitted over the public channel for information carrying and chaos synchronization. Accordingly, $s(\mathbf{y}, t) = g(t)y_1(t)$. In [1], $g(t)$ is selected as the product of a cosine signal and another system variable: $g(t) = A\cos(\omega t + \phi_0)x_3(t)$. When the above system runs in a chaotic masking configuration, $s(\mathbf{x}, t) = g(t)x_1(t) + i(t)$, where $i(t)$ is the plain-message signal.

In the above secure communication system, the secret key consists of the system parameters of the Lorenz system (σ, b, r) and the parameters of the modulating signal (ω, ϕ_0). Note that A should not be used as part of the key, since it does not influence the shape of the return map (just change the size). To facilitate the following discussion, without loss of generality, the parameters are fixed as the default parameters used in [1]: $\sigma = 16, r = 45.6, b = 4.0$, $A = 0.5, \omega = 1.5, \phi_0 = 0$. In the chaotic masking configuration, the plain-message signal is assumed to be $i(t) = 0.1\sin(t)$; in the chaotic switching configuration, the value of b switches between $b_0 = 4.0$ and $b_1 = 4.4$.

The purpose of using $g(t)$ is to frustrate return-map-based attacks, proposed



a) chaotic carrier: $x_1(t)$ b) chaotic masking: $x_1(t)+i(t)$ c) chaotic switching: $x_1(t)$



d) chaotic carrier: $g(t)x_1(t)$ e) chaotic masking: $g(t)x_1(t)+i(t)$ f) chaotic switching: $g(t)x_1(t)$

Fig. 1. Return maps (A_n vs B_n, the same thereinafter) reconstructed under different conditions.

previously in [16]. In common chaotic masking systems, $s(\mathbf{x},t) = x_1(t) + i(t)$, where $i(t)$ is the plain-message signal whose energy is much smaller than that of $x_1(t)$; and in common chaotic switching systems, $s(\mathbf{x},t) = x_1(t)$. Following [16], two return maps, A_n vs B_n and $(-C_n)$ vs $(-D_n)$, are defined as follows: $A_n = \frac{X_n+Y_n}{2}$, $B_n = X_n - Y_n$, $C_n = \frac{X_{n+1}+Y_n}{2}$, $D_n = Y_n - X_{n+1}$, where X_n and Y_m are the n-th (local) maxima and the m-th (local) minima of the transmitted signal, respectively. Since the A_n vs B_n map is identical to the $(-C_n)$ vs $(-D_n)$ map, in this paper only the former will be considered. Once an attacker gets the A_n vs B_n return map, he can use the attack method proposed in [16] to break both the chaotic masking and the chaotic switching systems.

In Fig. 1, some return maps reconstructed under different conditions are shown, from which it seems that the modulating function $g(t) = A\cos(\omega t + \phi_0)x_3(t)$ can effectively blur the return maps and so frustrate the attackers. However, as pointed out in [2,3], the modulating signal will introduce regular zero-crossing points in the transmitted signal $g(t)$, which makes it possible to distinguish the value of ω via spectral analysis [2] or autocorrelation analysis [3]. Also, it was shown in [4] that the power spectrum of $|s(\mathbf{x},t)|$ can be used to estimate the value of ω. Moreover, the value of ϕ_0 can be simultaneously obtained via autocorrelation analysis [3] or separately by detecting zero-crossing points at intervals equal to $\frac{\pi}{4\omega}$ [4]. A direct extraction of the binary plain-message signal from the transmitted signal has also been discussed in [4] for the chaotic switching configuration.

To resist the proposed attacks, a modified modulating signal was suggested in [3]: $g(t) = A(\cos(\omega t + \phi_0) + M)x_3(t)$, where $M > 1$. It was claimed that such a simple modification can eliminate the security defect of the original scheme and can also improve the synchronization performance. The default parameters of the modified scheme are identical to the ones used in the original Bu-Wang scheme, except $\phi_0 = 0.3$.

3 Breaking the scheme based on the modified modulation method

In this section, we show that the modified modulation method proposed in [3] can still be easily broken via a new attack, which is designed based on parameter estimation from the transmitted signal $s(\mathbf{x}, t)$. It will be shown that the proposed attack works for both chaotic masking and chaotic switching configurations. The most significant feature of the new attack is its independence of the zero-crossing points and its robustness with respect to different values of ω, ϕ_0 and M. This new attack is also able to break the original Bu-Wang scheme. This paper further points out that the modulation method itself is not very satisfactory from a cryptographical point of view, even if the modulating signal itself can be designed to be extremely secure against attacks.

3.1 Breaking the values of ω and ϕ_0

Assuming $g_0(t) = A(\cos(\omega t + \phi_0) + M)$, where $M > 1$ implies $g_0(t) > 0$. Thus, $|s(\mathbf{x}, t)| = |g_0(t)x_3(t)x_1(t)| = g_0(t)|x_3(t)x_1(t)|$, which means that the amplitude of $|x_3(t)x_1(t)|$ is periodically modulated by $g_0(t)$ at a fixed frequency ω. Consequently, there exists an embedded periodicity in the amplitude of $|s(\mathbf{x}, t)|$, just like the embedded periodicity in the zero-crossing points discussed in [2–4]. See Figs. 2a and b for a comparison of the waveforms of $s(\mathbf{x}, t)$, $|s(\mathbf{x}, t)|$ and $g_0(t)$, in the chaotic masking configuration with $i(t) = 0$, where the waveforms of $s(\mathbf{x}, t)$ and $|s(\mathbf{x}, t)|$ have been normalized to make the display clearer. One can see that the periodicity occurs as a clear envelop of the amplitude of $|s(\mathbf{x}, t)|$. After filtering $|s(\mathbf{x}, t)|$ with an averaging filter, one can dramatically refine the periodicity, as shown in Fig. 2c, where $\bar{s}(\mathbf{x}, t) = |s(\mathbf{x}, t)| \circledast f_a(t, 4)$ has also been normalized for a better display and the employed averaging filter $f_a(t, h)$ is defined as follows:

$$f_a(t, h) = \begin{cases} 1 - (|t|/h), & |t| \le h, \\ 0, & |t| > h. \end{cases} \tag{3}$$

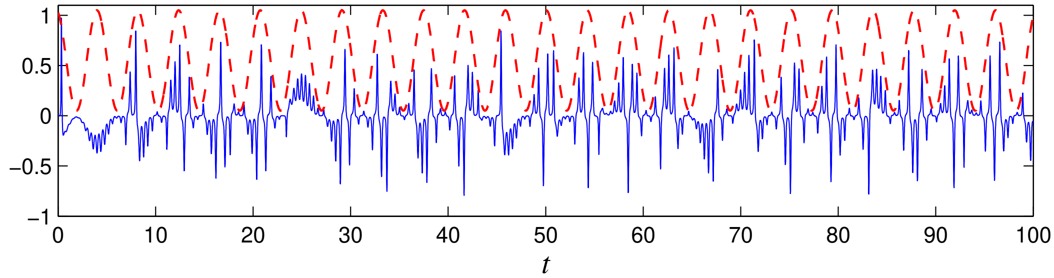
Here, $\circledast$ denotes the convolution operation and the value of h should be chosen close to ω to get a smooth filtered signal $\bar{s}(\mathbf{x}, t)$. For this purpose, a rough estimation of ω can be easily obtained from $|s(\mathbf{x},t)|$ as shown in Fig. 2b. Note that any other averaging filter may be used instead of the above triangular one.

From Fig. 2c, it is clear that one can easily get an accurate estimation of ω and ϕ_0, thanks to the global phase matching between $\bar{s}(\mathbf{x},t)$ and $g_0(t)$. Of course, some other estimation algorithms, such as the autocorrelation identification method proposed in [3], can be employed to further improve the accuracy of the estimation. When a plain-message signal is carried in the transmitted signal $s(\mathbf{x},t)$, the above estimation algorithm still works well as shown in Fig. 2d, which is attributed to the sufficiently small energy of the plain-message signal. Also, it is obvious that the above algorithm works well for the chaotic switching configuration (see Fig. 3 for a breaking result).

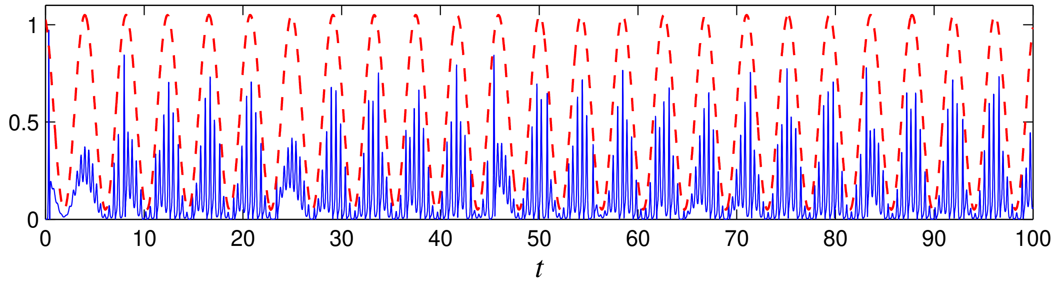
From the above discussion, it seems that the performance of the proposed estimation will gradually decrease as ω increases, since the averaging filter $f_a(t,h)$ may produce unsatisfactory results. This looks like a defect of the proposed attack. However, our experiments show that when ω is relatively large, it becomes possible to directly estimate the values of ω and ϕ_0 from $|s(\mathbf{x},t)|$ only (even without using average filtering). When $\omega = 13.6$ [1], for example, the waveforms of $|s(\mathbf{x},t)|$, $\bar{s}(\mathbf{x},t)$ and $g_0(t)$ are shown in Fig. 4, where the parameter of the averaging filter is chosen as $h = 0.2$. Note that the positions of local minima of $|s(\mathbf{x},t)|$ coincide very well with those of $g_0(t)$, although $\bar{s}(\mathbf{x},t)$ becomes less clear. This demonstrates that the proposed attack method is rather robust to different values of ω and ϕ_0.

It is worth mentioning that the value of ω can also be derived via spectral analysis: simply, calculating the power spectrum of $\bar{s}(\mathbf{x},t)$ and extracting the spectrum peak at the frequency ω. In fact, such a direct spectral analysis for estimating ω works even without using average filtering, as shown in [4]. This is because the following property of Lorenz-like chaotic systems [24, 37]: although $x_1(t)$ of the Lorenz system has a wideband spectrum as shown in Fig. 5a, the absolute-valued signal $|x_1(t)|$ has a prominent spectrum peak at an inherent frequency f_z, which is uniquely determined by the system parameters σ, b and r and is independent of the initial conditions. See Fig. 5b for the power spectrum of $|x_1(t)|$. This is also true for $x_2(t)$. For $x_3(t)$, the operation is even simpler: $x_3(t)$ itself has a prominent spectrum peak at the inherent frequency f_z. This implies that $|s(\mathbf{x},t)| = g_0(t)|x_3(t)x_1(t)|$ will have at least two prominent spectrum peaks, at f_z and $f_\omega = \dfrac{\omega}{2\pi}$ (Hz), respectively. See Figs. 5c and d for a comparison of the power spectra of $s(\mathbf{x},t)$ and $|s(\mathbf{x},t)|$. It can be

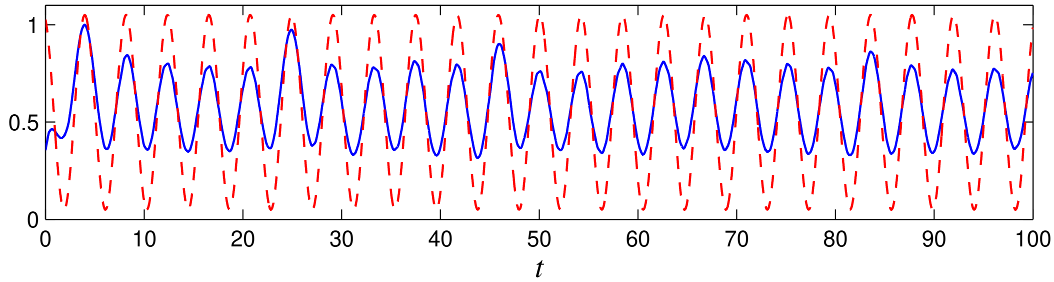
[1] This value is intentionally set to be identical to the inherent frequency $2\pi f_z$ of the Lorenz system, so as to emphasize on the robustness of the proposed attack. Some explanations of the inherent frequency of the Lorenz system are given in the following paragraphs.



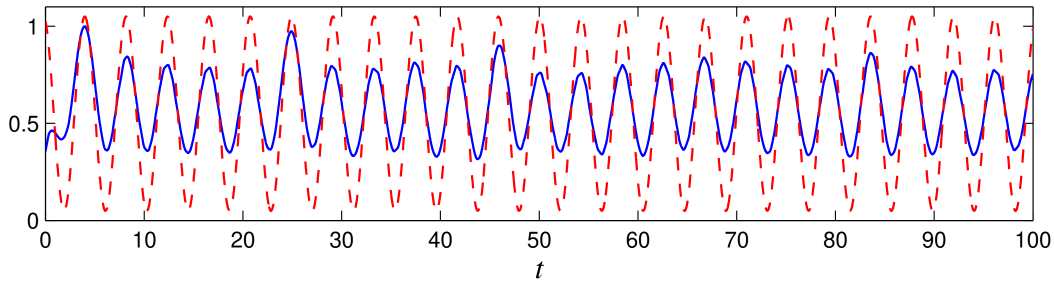
a) $s(\mathbf{x}, t)$ vs $g_0(t) = A(\cos(\omega t + \phi_0) + M)$, when $i(t) = 0$



b) $|s(\mathbf{x}, t)|$ vs $g_0(t) = A(\cos(\omega t + \phi_0) + M)$, when $i(t) = 0$



c) $\bar{s}(\mathbf{x}, t)$ vs $g_0(t) = A(\cos(\omega t + \phi_0) + M)$, when $i(t) = 0$



d) $\bar{s}(\mathbf{x}, t)$ vs $g_0(t) = A(\cos(\omega t + \phi_0) + M)$, when $i(t) = 0.1\sin(t)$

Fig. 2. Breaking the values of ω and ϕ_0 in the chaotic masking configuration, where and throughout the paper the dashed line is $g_0(t)$.

seen that the first prominent peak is at the frequency f_ω and the second at f_z. Apparently, ω can also be obtained from f_ω as $\omega = 2\pi f_\omega$. Following [24, 37], such an inherent frequency also exists in Chua and Rössler systems, so the same spectral analysis is also available for attacking these two chaotic systems widely-used in chaos-based secure communications. At present, it is not clear

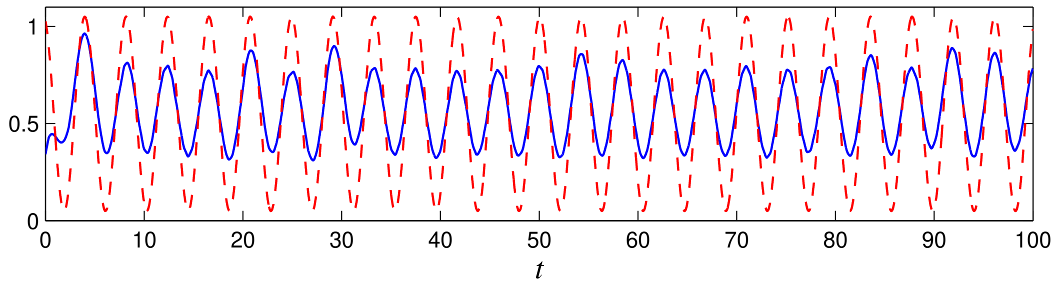
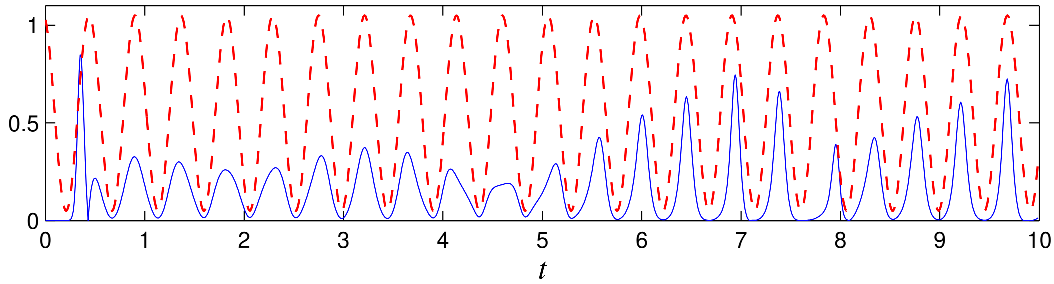
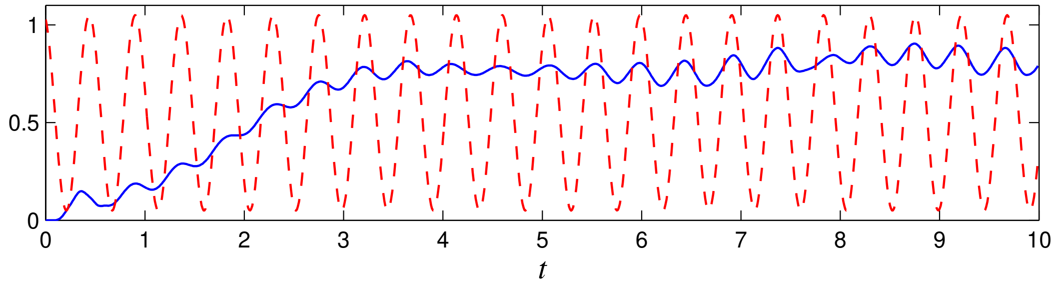


Fig. 3. Breaking the values of ω and ϕ_0 in the chaotic switching configuration: $\bar{s}(\mathbf{x}, t)$ vs $g_0(t) = A(\cos(\omega t + \phi_0) + M)$.



a) $|s(\mathbf{x}, t)|$ vs $g_0(t) = A(\cos(\omega t + \phi_0) + M)$



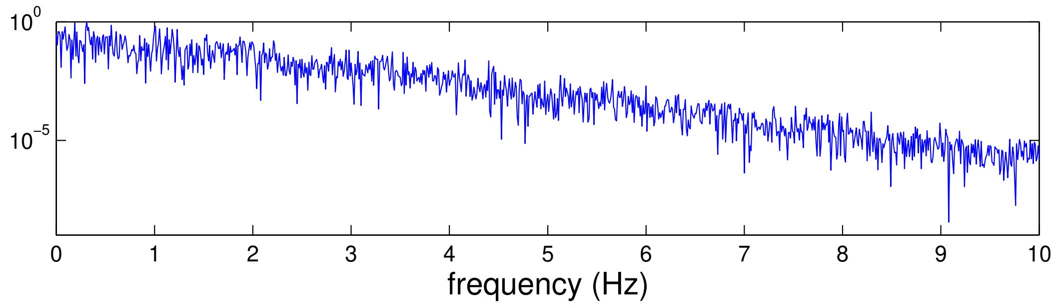
b) $\bar{s}(\mathbf{x}, t)$ vs $g_0(t) = A(\cos(\omega t + \phi_0) + M)$

Fig. 4. Breaking the values of ω and ϕ_0 in the chaotic masking configuration.

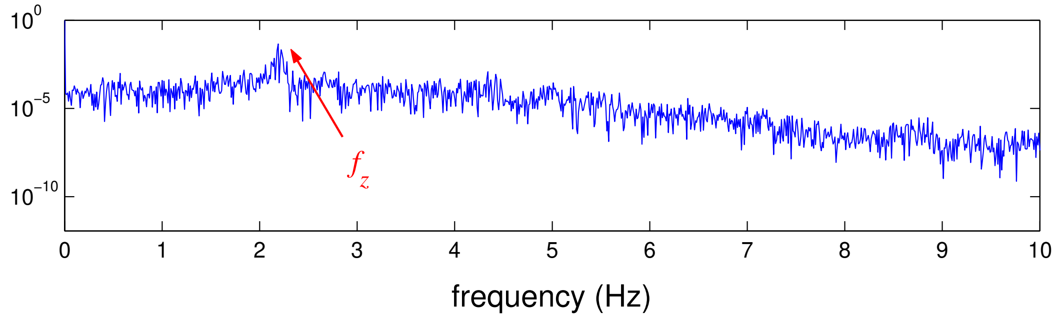
whether there exists any chaotic system that has no distinguishable inherent frequency.

Compared with the spatial attack proposed above, the existing spectral attack has some defects: 1) it cannot work well when $\omega \approx f_z$; 2) the value of ϕ_0 cannot be simultaneously estimated with ω in the frequency domain; 3) if there does not exist a strong DC component in $|x_3(t)x_1(t)|$ (e.g., if some other chaotic systems are also used), f_ω will not occur as a prominent peak in the power spectrum [2]; 4) when other chaotic systems are used instead of the Lorenz system, the inherent frequency may not exist (though unlikely) or the frequency f_ω might not be easily identified from the power spectra.

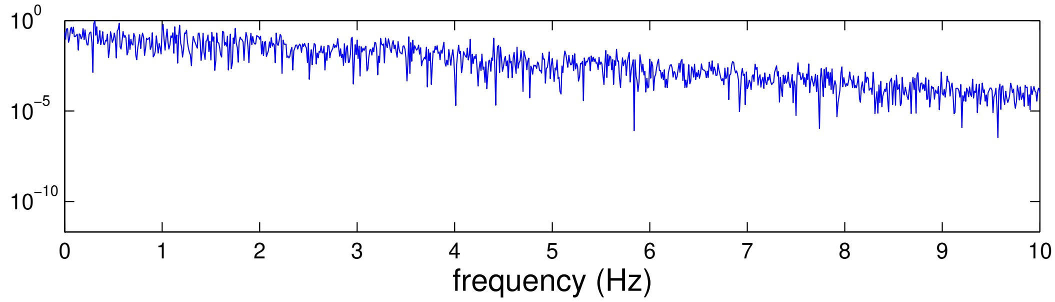
[2] Although $f_z \pm f_\omega$ always occur as two minor peaks, they are generally less distinguishable than the individual major peaks at f_ω and f_z, respectively.



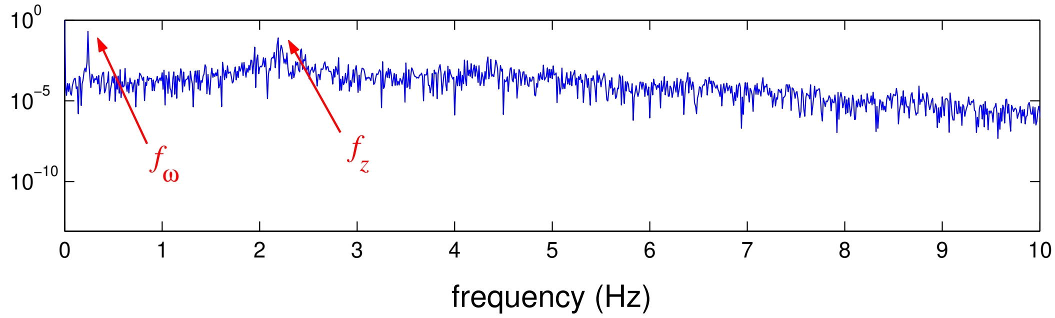
a) the relative power spectrum of $x_1(t)$



b) the relative power spectrum of $|x_1(t)|$



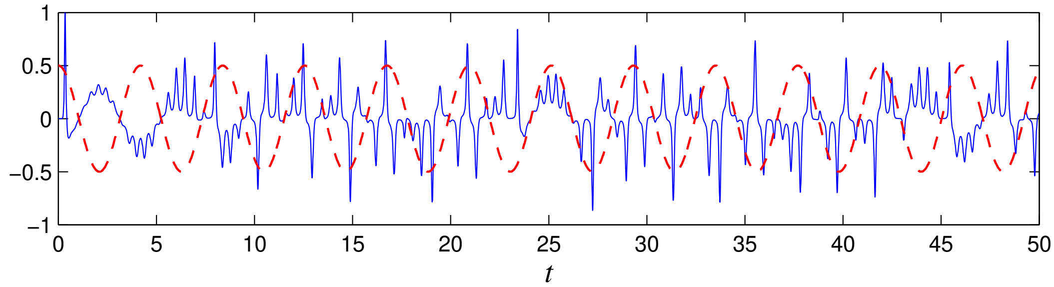
c) the relative power spectrum of $s(\mathbf{x}, t)$



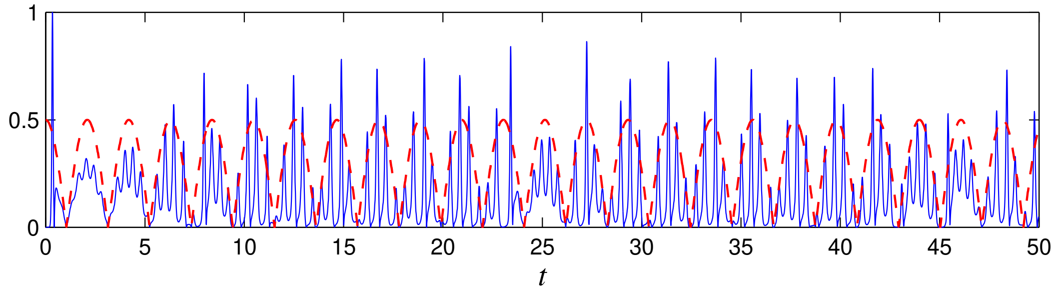
d) the relative power spectrum of $|s(\mathbf{x}, t)|$

Fig. 5. The relative power spectra of $x_1(t)$, $|x_1(t)|$, $s(\mathbf{x}, t)$ and $|s(\mathbf{x}, t)|$.

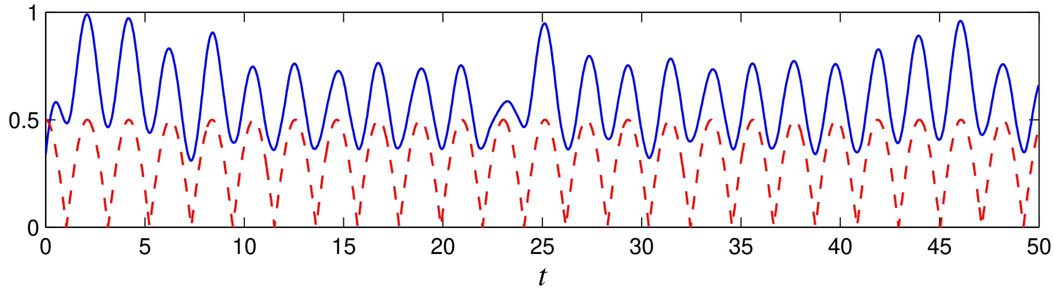
Finally, let us see how the proposed attack works on the original Bu-Wang scheme, where the condition is somewhat different since $g_0(t) = A\cos(\omega t + \phi_0)$ is not always positive. In that case, one has



a) $s(\mathbf{x},t)$ vs $g_0(t) = A\cos(\omega t + \phi_0)$



b) $|s(\mathbf{x},t)|$ vs $|g_0(t)| = |A\cos(\omega t + \phi_0)|$



c) $\bar{s}(\mathbf{x},t)$ vs $|g_0(t)| = |A\cos(\omega t + \phi_0)|$

Fig. 6. Breaking the values of ω and ϕ_0 in the original Bu-Wang scheme, where the dashed line denotes either $g_0(t)$ or $|g_0(t)|$.

$$\begin{aligned}|s(\mathbf{x},t)| &= A|\cos(\omega t+\phi_0)|\cdot|x_3(t)x_1(t)|\\ &= \frac{A}{\sqrt{2}}\cdot\sqrt{1+\cos(2\omega t+2\phi_0)}\cdot|x_3(t)x_1(t)|.\end{aligned}$$

Thus, with the above-described attack we will get 2ω and $2\phi_0$, which can be easily halved to obtain the values of ω and ϕ_0. In Fig. 6, the result of breaking the chaotic masking configuration with $i(t) = 0$ of the original Bu-Wang scheme is shown, where a different averaging filter, $f'_a(t,1)$, is used to make $\bar{s}(\mathbf{x},t)$ smoother:

$$f'_a(t,h) = \begin{cases} 1+\cos\left(\frac{2\pi t}{2h}\right), & |t| \le h \\ 0, & |t| > h. \end{cases} \tag{4}$$

From the above analysis, one can see that the attack proposed in this subsection can work for not only the original Bu-Wang scheme but also some other scheme if the modulating signal $g_0(t)$ is periodic (which need not be a simple sinusoidal or cosine signal). Therefore, an aperiodic or even chaotic modulating signal $g_0(t)$ has to be used to enhance the security. However, in this case the synchronization of the modulating signal itself becomes a new problem, which will be further discussed in Sec. 4.2.

3.2 Breaking the value of M

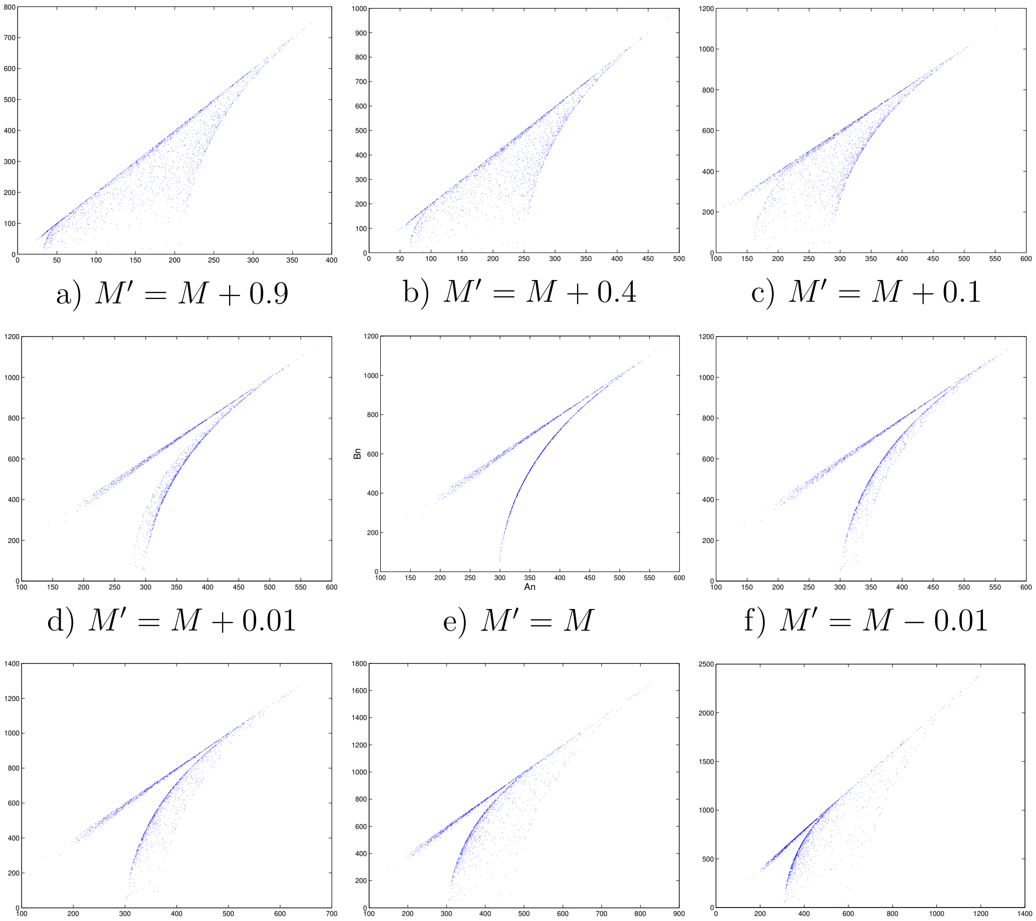
Once the values of ω and ϕ_0 are known, one can further break the value of M via an iterative process, and then get rid of the blurring effect on the return map by removing $g_0(t)$ from $s(\mathbf{x},t)$. To do so, define an auxiliary signal

$$\hat{s}(\mathbf{x},t) = \frac{s(\mathbf{x},t)}{\cos(\omega t+\phi_0)+M'} = \frac{\cos(\omega t+\phi_0)+M}{\cos(\omega t+\phi_0)+M'} \cdot Ax_3(t)x_1(t), \tag{5}$$

where M' is an estimation of M. Apparently, it is expected that the closer the value of M' is to M, the closer the signal $\hat{s}(\mathbf{x},t)$ is to $\hat{x}_{13}(t) = Ax_3(t)x_1(t)$. As a natural result, if M' changes from a value larger than M to a value smaller than M, i.e., $(M' > M) \to (M' < M)$, the behavior of $\hat{s}(\mathbf{x},t)$ will gradually go closer to $\hat{x}_{13}(t)$ and then turn gradually far away from $\hat{x}_{13}(t)$ once M' crosses the point $M' = M$. Therefore, there exists a global minimum at the point $M' = M$. The existence of such a global minimum can be easily checked by observing the reconstructed return map from $\hat{s}(\mathbf{x},t)$ or $|\hat{s}(\mathbf{x},t)|$. In Fig. 7, the return maps constructed from $|\hat{s}(\mathbf{x},t)|$ with respect to different values of M' are shown. The reason why $|\hat{s}(\mathbf{x},t)|$ is used is that this reconstructed return map has a simpler structure than the map reconstructed from $\hat{s}(\mathbf{x},t)$. The following features can be found from the maps:

(1) The closer the value of M' is to M, the thinner the branch width and the closer the return map reconstructed from $|\hat{s}(\mathbf{x},t)|$ is to the return map from $|\hat{x}_{13}(t)|$.
(2) The shape of the return map is related to the relation between M' and M: the two edges of the right-bottom branch are both clear when $M' > M$, while only the left edge is clear when $M' < M$. When M' is closer to M, the above characteristic is a little different: the right edge is (not so much) clearer than the left one when $M' > M$, while the left edge is *much* clearer than the right one when $M' < M$.

The first feature above means that one can approximately know the distance between M' and M by measuring the width of the right-bottom branch of the return map, and the second feature means that one can exactly know whether $M' > M$ or $M' < M$. Note that the branch width and the clearness of the two



a) $M' = M + 0.9$ b) $M' = M + 0.4$ c) $M' = M + 0.1$

d) $M' = M + 0.01$ e) $M' = M$ f) $M' = M - 0.01$

g) $M' = M - 0.02$ h) $M' = M - 0.04$ i) $M' = M - 0.06$

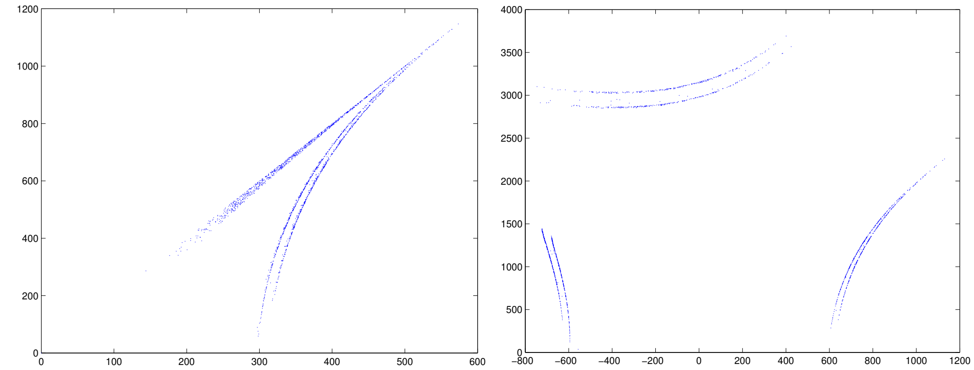
Fig. 7. Return maps reconstructed from $|\hat{s}(\mathbf{x}, t)|$ with different values of M'.

edges of the right-bottom branch can be quantitatively defined by different methods. For example, some algorithms can be used to extract the skeleton and the two edges of the branch, and then determine the two measures. In fact, in real attacks, an attacker can directly get a rough estimation of the measures by naked eye, and then *manually* carry out the following iterative algorithm for several times to find a sufficiently accurate estimation of M.

Given a lower bound $M_- < M$ and an upper bound $M_+ > M$, the iterative algorithm is as follows:

- *Step 1 – Initialization*: Set $i = 1$, $M_-^{(i)} = M_-$, $M_+^{(i)} = M_+$, and determine the number of iterative rounds, m (which is used to achieve a desired precision, as further explained later).
- *Step 2*: Among the following $n + 1$ values,

$$\left\{ M'_{i,j} = M_-^{(i)} + j \cdot \frac{\left(M_+^{(i)} - M_-^{(i)}\right)}{n} \right\}_{j=0}^{n}, \tag{6}$$



a) The return map reconstructed from $|\hat{s}(\mathbf{x}, t)|$

b) The return map reconstructed from $\hat{s}(\mathbf{x}, t)$

Fig. 8. The return maps reconstructed from $|\hat{s}(\mathbf{x}, t)|$ and $\hat{s}(\mathbf{x}, t)$ when $M' = M + 0.01$ in the chaotic switching configuration.

find two consecutive ones satisfying $M'_{i,j-1} < M$ and $M'_{i,j} > M$.

- *Step 3*: If $i \leq m$, then increase i by 1, set $M_-^{(i)} = M'_{i,j-1}$, $M_+^{(i)} = M'_{i,j}$, and goto Step 2; otherwise, stop the algorithm and output an estimation $\widetilde{M} = \frac{M'_{i,j-1} + M'_{i,j}}{2}$.

Apparently, the above iterative algorithm is rather similar to the numerical algorithms for solving algebraic equations for roots [38]. It can be easily deduced that the estimation accuracy after m rounds is $\left|\widetilde{M} - M\right| \leq \frac{M_+ - M_-}{2n^m}$. To achieve a predefined accuracy $\epsilon > 0$, it is required that $\frac{M_+ - M_-}{2n^m} \leq \epsilon$, and then one gets $m \geq \frac{\log_2(M_+ - M_-) - \log_2(2\epsilon)}{\log_2 n}$. Taking $m = \left\lceil \frac{\log_2(M_+ - M_-) - \log_2(2\epsilon)}{\log_2 n} \right\rceil$, the complexity of the above iterative algorithm is only $O(nm) = O\left(n \cdot \frac{\log_2(M_+ - M_-) - \log_2(2\epsilon)}{\log_2 n}\right)$, which is sufficiently small for an attacker to carry out the algorithm *manually*. When $n = 3$, the computing complexity is minimized, since the integer function $f(n) = \frac{n}{\log_2 n}$ $(n \geq 2)$ reaches the global minimum at $n = 3$.

Although in theory the above algorithm can get an arbitrarily accurate estimation by increasing the round number m, the distinguishability of the return map reconstructed from a finite set of data will be insufficient when M' is too close to M. Fortunately, in this case, the return map will be sufficiently clear for an intruder to carry out the return-map-based attacks. Figure 8 shows the return map reconstructed from $|\hat{s}(\mathbf{x}, t)|$ and $\hat{s}(\mathbf{x}, t)$ when $M' = M + 0.01$ in the chaotic switching configuration. It can be seen that such an accuracy is enough to break the 0/1-bits in the plain-message signal.

4 Discussions on the performance of the modulation-based methods

4.1 A cryptographical perspective

From a cryptographical point of view, a modulating signal is not an effective tool for enhancing the security of chaos-based secure communications, due to the dependence of the secret parameters of the chaotic systems on the secret parameters of the modulating signal. In fact, to break the whole chaotic cryptosystem via a brute-force attack, what an attacker should do is to exhaustively guess the secret parameters of the modulating signal, not all secret parameters (system parameters and modulating parameters). This means that the key space of the whole system is reduced to be $\max(K_m, K_c)$, i.e., to be K_m when $K_m > K_c$, where K_m is the key space of the modulating signal and K_c is the key space of the chaotic cryptosystem without modulation. However, for a good encryption algorithm, the key space should be always $K_m \times K_c$ [39]. To further enhance the modulation-based method, the modulating operation should be non-invertible, i.e., expressing $s(\mathbf{x}, t) = F(g_0(t), \mathbf{x})$, the function $F(\cdot, \cdot)$ should be non-invertible with respect to $\mathbf{x}$. However, with such a complicated modulating mechanism, the synchronization between the sender and the receiver may become much more difficult and be impossible under practical conditions (see the next subsection for further discussion on synchronization issues).

4.2 The synchronization performance

In [3], it was found that "almost no value can achieve the synchronization when $M = 0$", i.e., when the original Bu-Wang scheme [1] is used. Our experiments support this claim. Furthermore, when $g_0(t) = A\cos(\omega t + \phi_0)$, Matlab's numerical differentiation function, `ode45` used in this work, fails (i.e., enters into a dead lock) with a high probability for many values of c. This implies that the original synchronization scheme proposed in [1] is ill-behaved in most cases.

It can be confirmed that chaos synchronization of the modified scheme suggested in [3] can be achieved for a large range of values of c. This means that the modification in [3] really enhances the synchronization performance. For verification, a number of experiments were carried out to study the synchronization performance when $g_0(t)$ is replaced by some other functions. As a surprising result, it was found that even a non-negative noise signal can make the receiver system synchronized well with the sender system. Figure

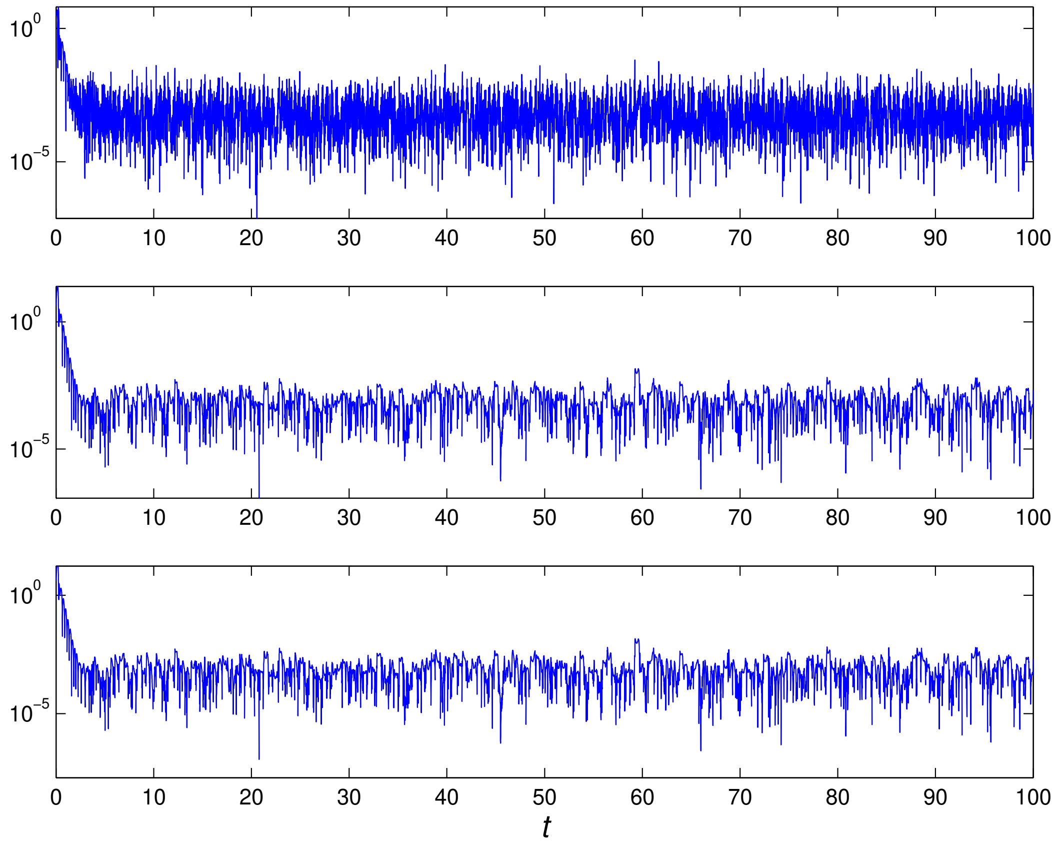


Fig. 9. The synchronization errors of the three system variables when $g_0(t) = \mathrm{rand}(t) \in (0, 1)$: $|y_1(t) - x_1(t)|$, $|y_2(t) - x_2(t)|$ and $|y_3(t) - x_3(t)|$ (from top to bottom).

9 shows the experimental result when $g_0(t) = \mathrm{rand}(t) \in (0, 1)$ and $c = 10$. Some other signals were also tested, for example, $g_0(t) = t^2 \bmod 1$ and $g_0(t) = (t\cos(\omega t + \phi_0)) \bmod 1$, and it was found that chaos synchronization is achieved in both cases. At present, the reason behind this unexpected synchronization phenomenon is still not clear.

5 Conclusion

In [1], a simple modulation-based method was proposed to improve the security of chaos-based secure communications against return-map-based attacks. Soon this scheme was independently cryptanalyzed in [2–4] via different attacks. Then, in [3], a modified modulation-based scheme was proposed to enhance the security of the original one. This paper proposes a new attack to break the new modified scheme. Compared with previous attacks, the proposed attack is more powerful and can also break the original scheme in [1]. Based on all analysis and experimental results, the conclusion is that, from a cryptographical point of view, the security of this class of modulation-based scheme is not satisfactory for secure communications.

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