Research on Non-linear Dynamic Systems Employing Color Space

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Abstract This paper introduces a new method of research on evolvement of non-linear dynamic system employing color space, such as fractal sets and chaos systems. Contrast to traditionary methods, it has more intuitionistic display and can express more information of systems, and we can observe more hidden features of dynamic systems by employing it. And you can also obtain the bifurcation figures, final iterative figures. This method is mainly used for researches on those iterative systems whose dimesion is lower than 3. It also can be used for researches of low dimensions mapping characteristics of those high-dimension dynamic systems. This paper gives three samples on Mandelbrot/Julia set, Logistic mapping and Hénon trajectory.

Keywords dynamics, color space, fractal, chaos, bifurcation, Mandelbrot, Julia, Logistic

1. Introduction

1.1 Non-linear science, dynamics, fractal and chaos

Non-linear science is a discipline which has grown up on the base of the research on the sub-disciplines that have the characteristics of non-linearity. The main task of this discipline is to research the dynamic characteristics of non-linear dynamics system. It deals with many domains such as physics, chemic, life sciences, astronomy, material science, etc. At the same time, it boosts up the development of many new subjects such as dissipation structure theory, mutation theory, fractal geometry, self-organizing theory and so on. Fractal and chaos are the two mostly active embranchments of the non-linear science. Fractal emphasizes the nature of non-linear system using the description from the mathematic point of view, including self-similarity, the Hausdroff dimenson larger than topology dimenson and etc. While chaos system stress on descripting the physical states in the evolvement of non-linear system, and descripting the characters of the systems which are nonperiodic, stochastic-like, but have certain mathematic evolvement equations. In fact, fractals and chaos are two main features of non-linear system. Researches on them have great meanings on the research of fractal sets and chaos system of representative non-linear system and the revelation of the dynamic essence of non-linear system.

The research on fractal sets and chaos system has been the keystones of research on the non-linear systems since its rise. Fractal has formed special discipline: fractal geometry, chaos-related researches have involved with divarication theory, topology, dissipation structure and etc. Fractal geometry was formed on the epochmaking work of Mandelbrot in the 70th,he published his first writings on fractal geometry *Fractal: Form, Chance and Dimension*.He successfully made an inaugurative research which made the affiliation between the past research on fractal and many practical issues. Then the fractal geometry became an independent subject. In the past 20 years, the scope of mathematic theory of fractal sets and its practical research have been greatly developed, and in fact fractal and chaos have become the symbol of non-linear dynamics research. An important theoretical problem in fractal geometry is the generation of various typical fractal sets and analysis of their structures and features, the most typical fractal sets include Mandelbrot set, Julia set, they both come from the $z(k+1)=z(k)^2+c$ dynamic system on compound plane. This article will use the method of color space to make a qualitative analysis of these two fractal sets. One typical example of chaos system is the Logistic mapping $x(k+1)=rx(k)(1-x(k))$. Feigenbaum made a particular research firstly, and discovered Feigenbuam constant of the bifurcation phenomena on system evolving period, this became the important milestone of chaos research, this paper gives an analysis of this typical example.

1.2 Color theory and color space

Color theory is a discipline about human color perception, quantitative/qualitative measurement and analysis of color, etc. Color is a apperceive amplitude of human being, its physiology foundation is that human eyes have cone-shaped cells which can apperceive the three basic colors: Red, Green and Blue. The human perception of colors is decided by the stimulation of these three types of cells produced by the energy composition distribution of incidence light. And The human perception of brightness is decided by two kind of human eyes cells: pole-shaped cells and cone-shaped cells, the former mainly produce dark vision, while the latter mostly the bright vision.

To make a quantitative description of human color perception, now some special color systems have been developed. Color systems can be divided into two large categories:color mixing systems and color appearnce systems. Color mixing systems make use of the calculation of the necessary color light amplitude that can match some certain color using the light color mixing experiments. Color appearnce system is a kind of color system based on the appearance of certain standard colored objects (such as color cards). Typical color mixing system is CIE color mixing system, typical color appearnce systems are Munsell color system, Ostwald color system, DIN color system, OSA color system and so on, which are all color appearnce systems adopting color cards. For any color systems there must be three independent variables to describe one kind of color. Munsell Color system calls thoes three variables as value, hue and chroma. CIE color system uses the three RGB stimuluses or XYZ stimuluses that have been decided by experiments. Munsell color system use cylinder coordinates to locate color cards, the ordinate stands for value, the circle direction means hue and the radius direction means chroma (white, grey and black have the zero value of chroma). CIE color systems include RGB color system and XYZ color system, the latter is more popular. For convenience, every chromaticity point is mostly denoted by Y and the chromaticity coordinates (x, y), which is determined by the point coordinates of intersection of chromaticity vector and X+Y+Z=1 plane. The color that can be perceived by human eyes in the plane X+Y+Z=1 is a hoof-shaped, and it is called CIExy chromaticity diagram. We can see that any color systems can hold a three-dimensional color space (it is called a color solid in Munsell color system).

The color composition method used in digital computer is tricolor mixing, the store of colors should be discreted. The common method is to quantify the three value (RGB) into 256 steps,then stores one color using three bytes(R,G,B). Actually the number of chromaticity point obtained by this method is quite discreted. The chromaticity range is a discrete subset of consecutive color space, so in the rough we can regard the color space digitalized by computer as a discrete chromaticity cube space whose coordinates are the three value of RGB.

2. How to express fractal and chaos employing color space?

Because the astringency of fractal sets and the chaos system, their evolvement is localized in finite space. For the system whose dimension is not larger than three, we can use color space to denote the finite sub-space that the system is in. As there is a one-to-one mapping relation between the chromaticity and the chromaticity coordinate of every point in the sub-space, we can see the simulative process of system evolvement clearly in this sub space, while the chromaticity of each point is corresponding to the real coordinate of system evolvement. To observe the evolvement sequence in color space, we can observe the full process of the system evolvement.

In fact, we must determine which color space and which one-to-one mapping are employed. What's more, we should consider how to design to get more convenient display of system observation.

Because the biggest dimension of color space is three, theoretically we can use that method to research one-dimensional, two-dimensioal and three-dimensional systems. For the systems whose dimension is larger than three we can reduce their dimension and study their projection characteristics in the low-dimensional space, then we can analyze those high-dimensional systems.

For one-dimensional systems we can simply use one coordinate of color system to demarcate the system evolvement. Commonly we select the relative independent brightness axis (value axis in Munsell), the corresponding chromaticity point can be any point in chromaticity diagram, In RGB color system we choose R=G=B. For two-dimensional systems we can adopt some certain chromaticity diagram plane, and for three-dimensional system we can use the full color space of color system. For discrete system we can use the forementioned discrete chroma cube that can be stored in computer.

Here we mainly give out two actual examples of two-dimensional systems, one example of one-dimensional system. We won' t give a instance of three-dimensional system. And what should be noticed is the constringent space should be a sub-space of the color space employed. To make full use of color space we may transform color space to make the difference between the color space and the constringency space as conceivably small as possible.

3. Some Instances of research on fractal sets and chaos system using color space

3.1 Compound dynamic iterative system $z(k+1) = z(k)^2 + c$ --Mandelbrot/Julia set

The mathematical analysis of compound dynamic iterative system $z(k+1) = z(k)^2 + c$

indicates such a fact that if $|z| > 2$ then this point will be emanative. So actually Mandelbrot sets and Julia sets are both confined to the circle $|z| = 2$, namely the constringency space of the system is a sub-set of $|z| = 2$. The chromaticity planes that can be selected here are CIE chromaticity diagram plane and Munsell identical value plane.

We make the chosen chromaticity plane as finite space of system iteration, and do compound iteration $z(k+1) = z(k)^2 + c$ in it. The color of each iteration point during the iteration process means which point it has been iterated to, for all of the emanative points, simply give it a black color to know from other constringency points.

Since the CIE chromaticity plane is not a circle but a hoof-shaped, we can make a correction to the CIE chromaticity plane. To simplify problem, we select CIE-RGB color system whose chromaticity plane is triangle. Through dividing RGB triangle into three sections we can find out one method from RGB triangle to RGB circle. To obtain more convenient display we should make some more transforms to RGB circle chromaticity plane, such as negative-chromiticity operation and radius correction.

For Mandelbrot set, we commonly adopt the method of analysing final state figure. Most Mandelbrot set figure is drawn by filled color in the emanative part and filled black in the convergence part. However, the convergence part of system has more importance than the emanative part. So here we filled black in the emanative part and filled color in the convergence part. From such figure we can draw more conclusion about Mandelbrot set. We can find bifurcation phenomena and Feigenbaum constant from such figure scene. We can find N-period phenomena in Mandelbrot set. We list some results that are obtained in our rearch on Mandelbrot sets in figure-1 to figure-4.

We know that Mandelbrot set is the index of the Julia sets, we can select parameter c of different location in Mandelbrot set to study the characteristcs of the filled Julia sets. Figure-5 enumerates six filled Julia sets we have actually got (because the non-connective Julia sets can't be seen in the final iterative figure, there are all connective Julia sets). Inside Julia sets the iterative features are decided by the location of parameter c in Mandelbrot sets, if the location of c in the system is a N periods movement in Mandelbrot sets then the corresponding Julia incarcerated parts is also a N periods movement when the iteration number $n \to \infty$. In fact, the larger N is, the closer the parameter c will be to the border of Mandelbrot set, the more fragmental the obtained Julia set will be, the weaker its connectivity will be. And when c goes from the convergence part into the divergence area of Mandelbrot set, the obtained Julia set will become a non-connective discrete points set.

3.2 Two-dimensional Poincaré section plane –Hénon trajectory as example

Hénon trajectory is a trajectory that is represented in the form of Poincaré section which satisfy the following iterative equation formula-1. The color of certain point represents the location that the trajectory traverses the section at the interative time. Once the initial location is given, then the evolvement of the system trajectory is determined. Figure-6 shows the full view and part of it. We select the same color system as above in figure-6.

$$x(k+1) = x(k)\cos a - [y(k) - x(k)^2]\sin a$$
$$y(k+1) = x(k)\sin a + [y(k) - x(k)^2]\cos a \quad \text{formula-1}$$

3.3 One-dimensional chaos system-Logistic mapping

Logistic mapping is a simple one-dimensional system that can produce chaos and is sufficiently attentioned early in the study of chaos systems. It is used to describe the mutation of species in the ecosystem. It makes people realize the complexity hiding in simplicity. Its description equation is $x(k+1) = rx(k)(1 - x(k))$. As mentioned before, we can use brightness as description axis, select chromaticity point (R, G, B)=(1/3,1/3,1/3). Actually the brightness here is represented as greyscale in computer. We quantify the iteration result to 256-step greyscale, use abscissa to represent parameter r, and the ordinate independent variable x. We can get the greyscale iterative figures of Logistic mapping. Fixing the independent variable x, change parameter r only we can get the typical bifurcation figures. Please see also figure-7 to figure-10.

4. Conclusion and Summary

Through the analysis of several typical examples above, we can see the great potential for applying color space to study non-linear dynamics systems. In fact we can also use color space to study more generic three-dimensional systems, such as the typical Lorenz atmospheric dynamics system, it can also be projected to two-dimensional plane to be studied, we will not give any examples here for the reason of paper length.

Actually we have great flexibleness when we use color space to describe the non-linear dynamics systems. We can freely decide which color system is selected, how to make the transformation of color space, how to use color space to describe the evolvement process of the system and how to use corresponding method to depict the different features of different systems. We will find more skillful techniques and methods to solve those problems. We believe that the research method of color space will be useful in the future study of non-linear dynamics systems.

[Appdenix]-Figures

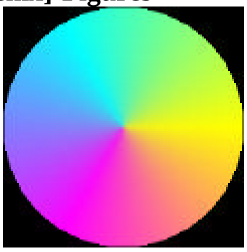
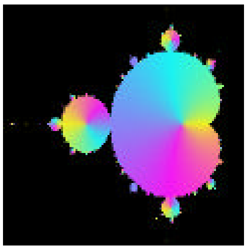

Figure-1 RGB chromaticity Circle
Figure-2 Mandelbrot set
after negative-chromiticity operation,radius correction
iteration number=100

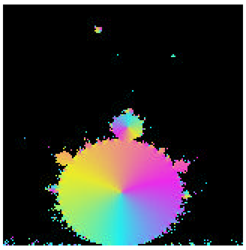

Figure-3 Local Mandelbrot set

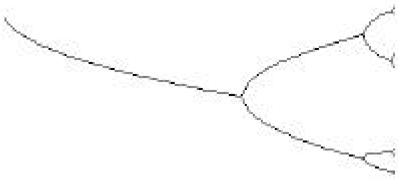

Figure-4 Bifurcation figure of Mandelbrot set
3-period series local part complex
coordinate range: (0.25,0) to (-1.4,0)

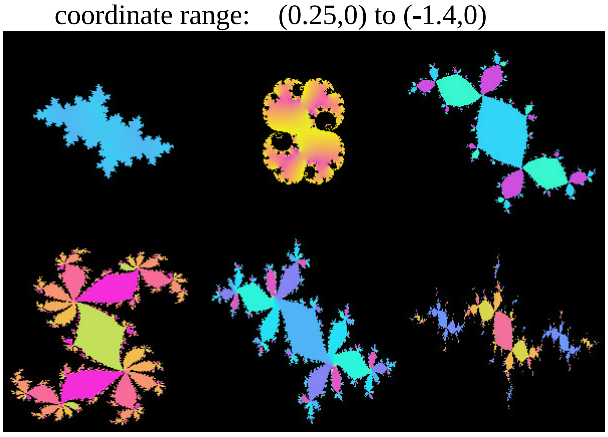

Figure-5 six Julia connective sets obtained

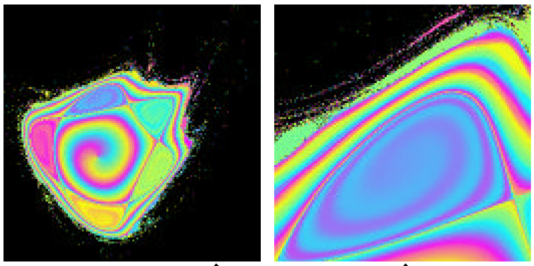

Figure-6 the Poincaré section of Hénon trajectory and its local part ($a = 77.16°$)

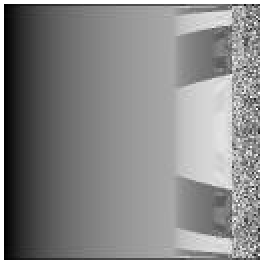
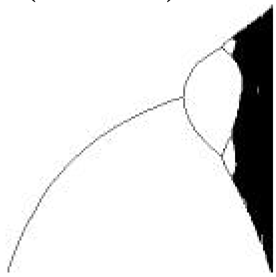

Figure-7 Logistic mapping interation figure
Figure-8 Bifurcation figure $x = 0.5$
$r \in (0,4)$ (from Figure-7)


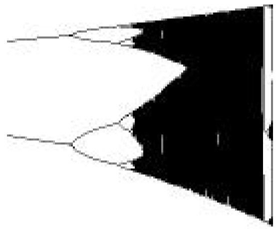

Figure-9 Logistic mapping interation figure
Figure-10 Bifurcation figure when $x = 0.54$
$r \in (3.31461, 3.85393)$ (Figure-9)

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