

# Cryptanalysis of the Convex Hull Click Human Identification Protocol

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# What's Inside?

## Chapters

1. Introduction to Human Identification Protocols
2. The Convex Hull Click Protocol
3. Proposed Attacks on Convex Hull Click
4. Conclusion

## Questions

# Chapter 1

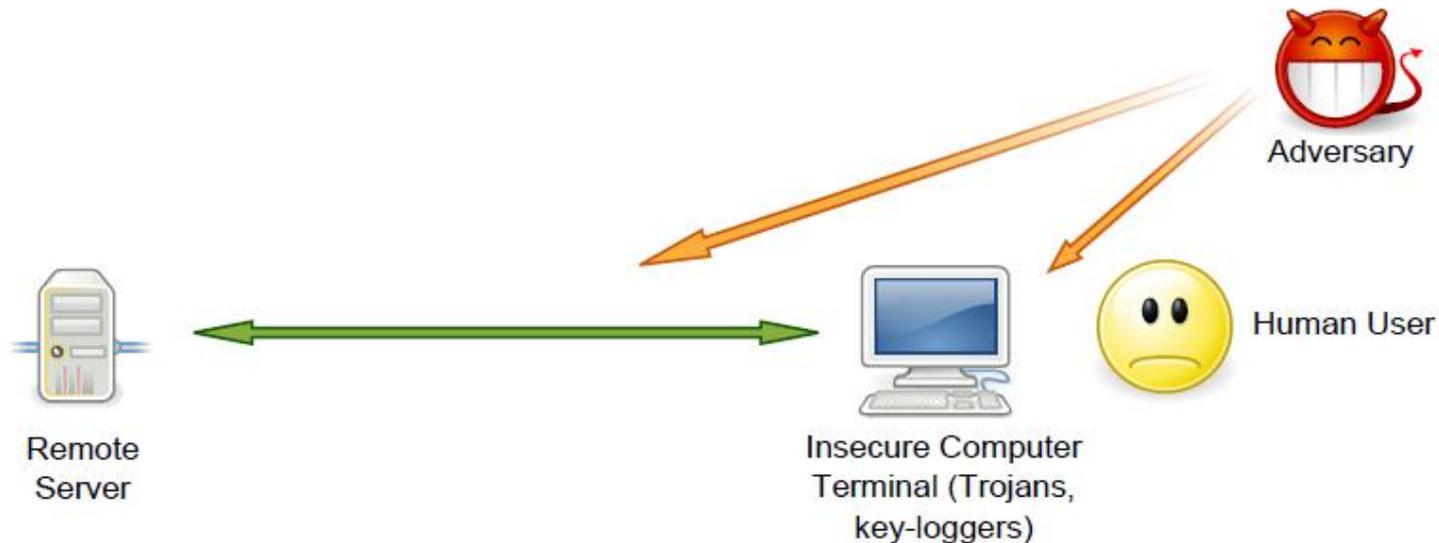
## Introduction to Human Identification Protocols (HIPs)

# The Problem with Human Identification

How to identify a human user when

Using an insecure terminal

An eavesdropper is watching



The model was first conceived by Matsumoto and Imai (1991)

# How Good are Traditional Solutions?

## Passwords?

Can be guessed or key-logged, etc

## PINs?

Short and often easy to guess (birthdays)

Susceptible to shoulder surfing

## Trusted hardware or biometrics?

Privacy concerns

Adversary can corrupt the equipment

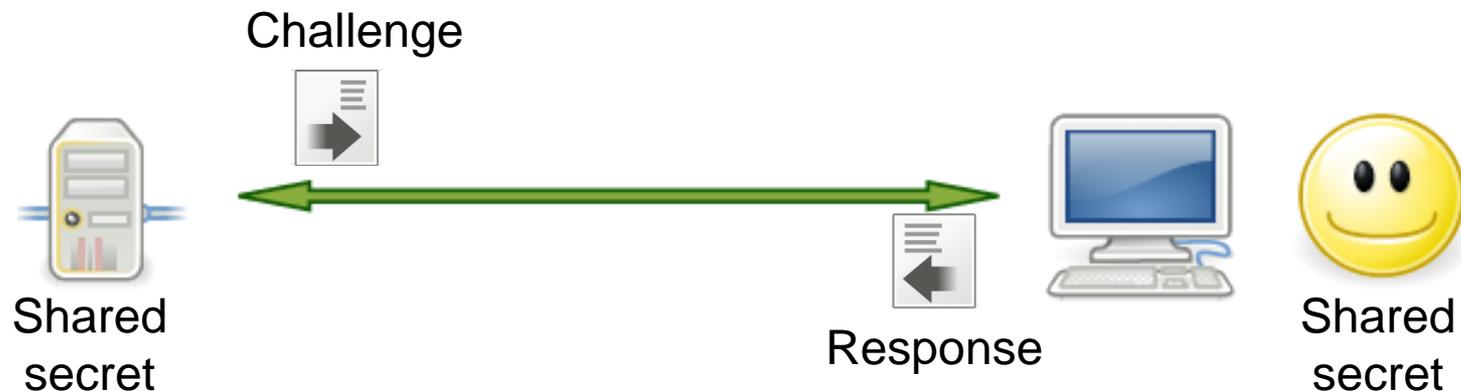
# A Possible Solution

## Challenge-response type authentication

User  $H$  and computer server  $C$  share a secret

$C$  sends a challenge to  $H$

$H$  sends a response as a function of the challenge and secret



# What is a Human Identification Protocol (HIP)?

A challenge-response protocol in which the prover is a human

## General Structure of an HIP

Setup Phase: establish a shared secret

Authentication Phase: challenge-response messages

# How about Security?

No one should be able to impersonate  $H$ , even after observing successful authentication sessions

Note: the adversary can see challenge-response “pairs”

We consider passive adversaries

How can I impersonate  $H$ ?



# ... And Usability?

As adversary can see everything

Any calculations have to be done mentally

Calculations should be easy to carry out

Authentication time should be as low as possible

# The Challenge

Find an identification protocol

Given a challenge and a secret, it is easy to compute response

Anyone observing challenges and responses cannot learn much about the secret



# Chapter 2

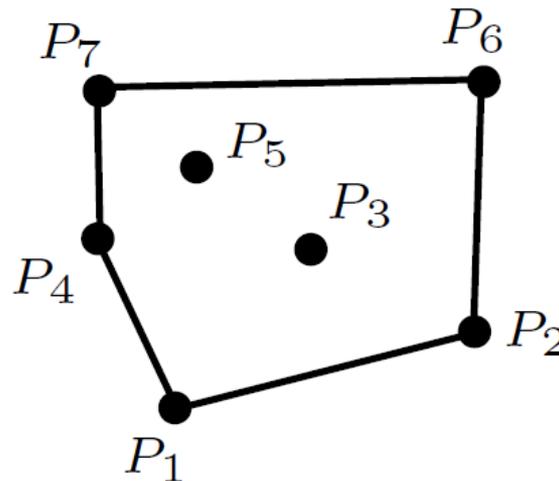
## The Convex Hull Click (CHC) Human Identification Protocol

# Convex Hull Click Protocol

A graphical authentication protocol proposed by Sobrado and Birget (2002)

More detailed work is done by Wiedenbeck et al. (2006)

Based on forming convex hulls of secret icons



# The Protocol

## Setup Phase

Given a large pool of graphical icons

Select  $k$  icons as secret icons (say  $k = 5$ )



# The Protocol

## Authentication Phase

Mentally form a convex hull of any 3 secret icons

Click a random point inside

Run 10 times to avoid random guessing



# Protocol Parameters

## Default

Total icons  $n = 112$

Icons displayed  $m = 43$  to  $112$

83 on average

Number of secret icons  $k = 5$

## High security

$n = 500$ ,  $m = 200$ ,  $k = 12$

Convex hull formed from any 3 secret icons

# Why Convex Hull Click?

Not just a graphical password scheme

An instance of a geometric problem being used for human identification protocol

Much like finding hard mathematical problems for cryptography!

# Chapter 3

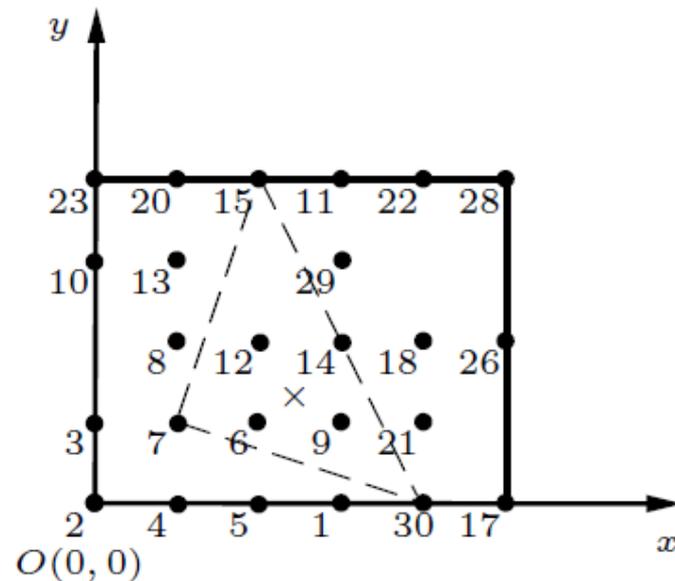
## Proposed Attacks

# Simplifications

## Assume

Fixed  $m$  (number of displayed icons)

Icons are lattice points with integer labels



# Attack 1

## Idea

At least 3 secret icons should be present

Difference in probability

Secret vs. non-secret icons

## The Attack

Rank icons according to the frequency of appearance

Output top  $k$  icons

# Results

With high probability top  $k$  icons are the secret icons

**Table 1.** Simulation Results for Attack 1

$n$	$m$	$k$	$r$	Average Number of Secret Labels	Probability of Finding all $k$ Secret Labels
112	70	5	100	4.6	0.622
500	200	12	100	11.4	0.554
112	90	5	100	0.1	0.000
500	313	12	100	0.2	0.000

# How to Avoid Attack 1?

The two probabilities should be same

Rule:

$$\frac{2km}{k+3} = n$$

# New System Parameter Values

If  $n = 112$  and  $k = 5$

$m$  should be 90

If  $n = 500$  and  $k = 12$

$m$  should be 313

Perhaps too high for comfort!

If  $m = 200$  and  $k = 12$

$n$  should be 320

# Attack 2

## Idea

Brute force attack works with  $O(n \text{ choose } k)$

But user only uses 3 secret icons

There could be an attack with  $O(n \text{ choose } 3)$

# The Attack

Given a challenge and a clicked point  $P$

Test Set  $C$

Find all 3-combinations of icons that contain point  $P$

Make frequency lists

For 3-combinations in  $C$

For individual icons  $L$  in  $C$

# The Attack

For  $r$  challenge-response pairs

Increment the frequency of each 3-combination in  $C$  if it contains the response points

Increment the frequency of labels

Threshold

Find all 3-combinations in  $C$  that have frequency higher than threshold

Output the most frequent label in the thresholded set

# Results

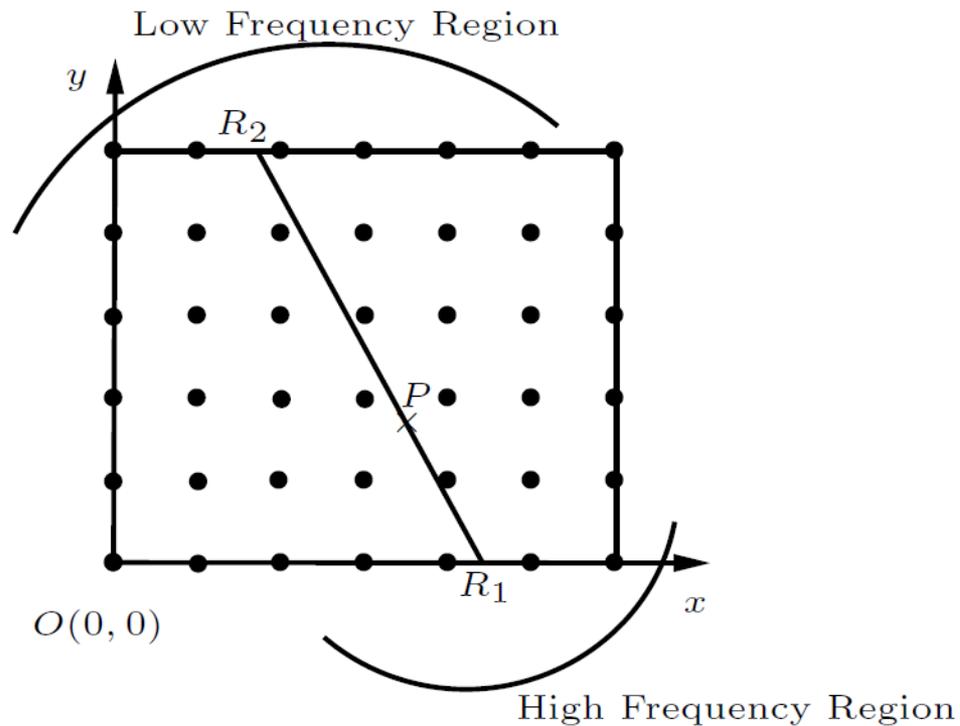
With high probability, output is one of the secret icons

**Table 4.** Output of Attack 2

Simulation Number	$n$	$m$	$k$	pairs	Secret Appeared	Average Threshold
1	112	90	5	20	$64/100 = 0.64$	6.4
2				30	$76/100 = 0.76$	7.8
3				50	$88/100 = 0.88$	10.9
4	160	100	12	20	$35/100 = 0.35$	4.8
5				30	$40/100 = 0.40$	5.6
6				50	$48/100 = 0.48$	7.2

# Why Does Attack 2 Work?

Points around  $PR_1$  can form more 3-combinations containing  $P$



# Why Does Attack 2 Work?

With high probability one of the secret icons is in the high frequency region

Because the convex hull formed is a triangle

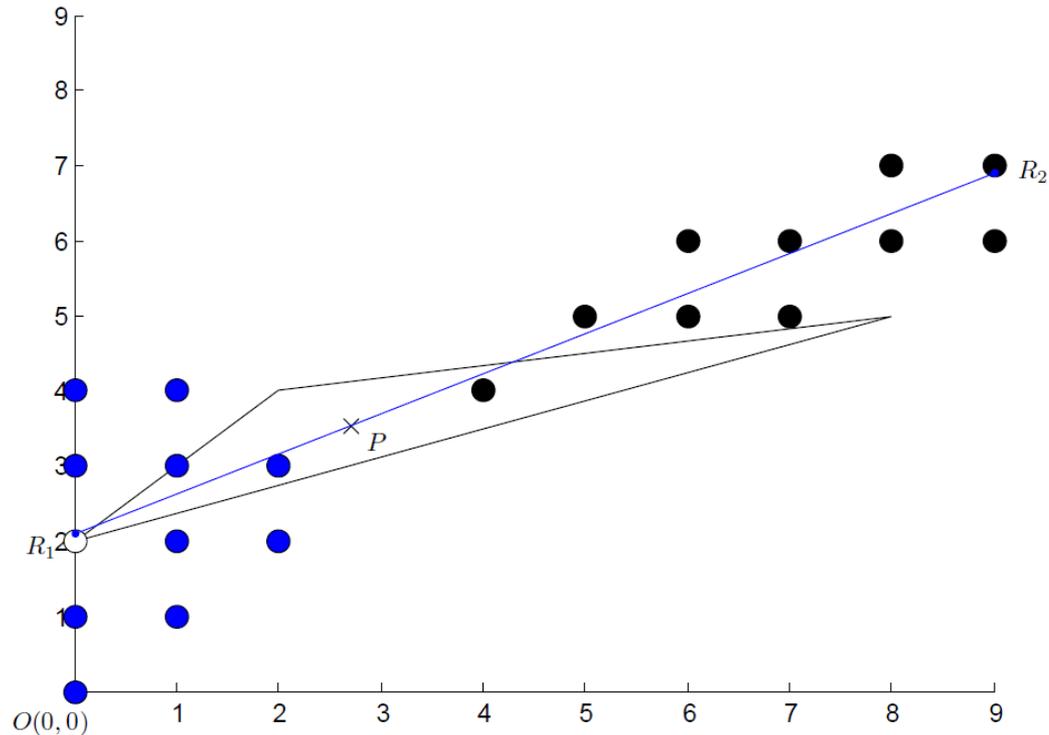
# Simulation

## Legends

Blue dots are icons with highest frequencies

Black dots are icons with lowest frequencies

White dot is one of the secret icons



# Improved Attack 2

## Idea

Given  $r$  challenge-response pairs

Choose the challenge-response pair as the test set

That has the clicked point closest to the rectangle

Call it Chosen Test Set Attack

# Results

## Considerable improvement

**Table 5.** Output of the Chosen Test Set Attack

Simulation Number	$n$	$m$	$k$	pairs	Secret Appeared	Sessions
1	112	90	5	20	$77/100 = 0.77$	10
2				30	$83/100 = 0.83$	14
3				50	$95/100 = 0.95$	20
4	160	100	12	20	$50/100 = 0.50$	77
5				30	$67/100 = 0.67$	86
6				50	$78/100 = 0.78$	123
7	320	200	12	20	$46/100 = 0.46$	83
8				30	$46/100 = 0.46$	125
9				50	$59/100 = 0.59$	163
10	357	200	25	20	$35/100 = 0.35$	330

# Impersonation

Attack 2 can be used to impersonate a user

Adversary does not need to find all secret icons

Suppose adversary has 3 out of 5 secret icons

# Impersonation Stage

If a challenge contains all 3

Click a random point within the convex hull

If a challenge contains only 2

Click on the line joining the two

If a challenge contains only 1

Click on the icon

Effective security is  $k - 2$  instead of  $k$

# Results

Suppose adversary uses Chosen Test Set Attack to find  $t$  icons

Probability that all  $t$  are secret labels and adversary succeeds in impersonating:

$$n = 112, m = 90, k = 5, r = 30, t = 2$$

$$\text{Probability} = 0.59$$

$$n = 160, m = 100, k = 12, r = 50, t = 3$$

$$\text{Probability} = 0.27$$

$$n = 320, m = 200, k = 12, r = 50, t = 2$$

$$\text{Probability} = 0.15$$

# Insecurity of CHC

Probability of random clicks being successful is less than

$$2^{-10} \approx 0.00098$$

Convex Hull Click is not secure against an eavesdropping adversary

Can impersonate user with non-negligible probability after observing only 5 to 15 sessions

# Discussion and Conclusion

Problem is an inherent one

Structure of convex hulls leaks information

It is an interesting direction to find new geometric problems for human identification

Or find improvements to CHC (if possible)

# References

- Matsumoto, T & Imai, H 1991, 'Human Identification through Insecure Channel', In *Advances in Cryptology - EUROCRYPT '91*, volume 547 of *Lecture Notes in Computer Science*, pages 409-421, Springer-Verlag.
- Sobrado, L & Birget, JC 2002, 'Graphical Passwords', *The Rutgers Scholar*, volume 4, Available online at <http://rutgersscholar.rutgers.edu/volume04/sobrbirg/sobrbirg.htm>.
- Wiedenbeck, S, Waters, J, Sobrado, L & Birget, JC 2006, 'Design and Evaluation of a Shoulder-Surfing Resistant Graphical Password Scheme', In *Proceedings of the Working Conference on Advanced Visual Interfaces - AVI '06*, pages 177-184, ACM.

# Questions

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