

# Breaking Randomized Linear Generation Functions based Virtual Password System

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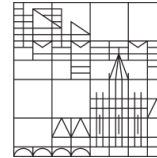
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26 May, 2010

- The problem/system under study
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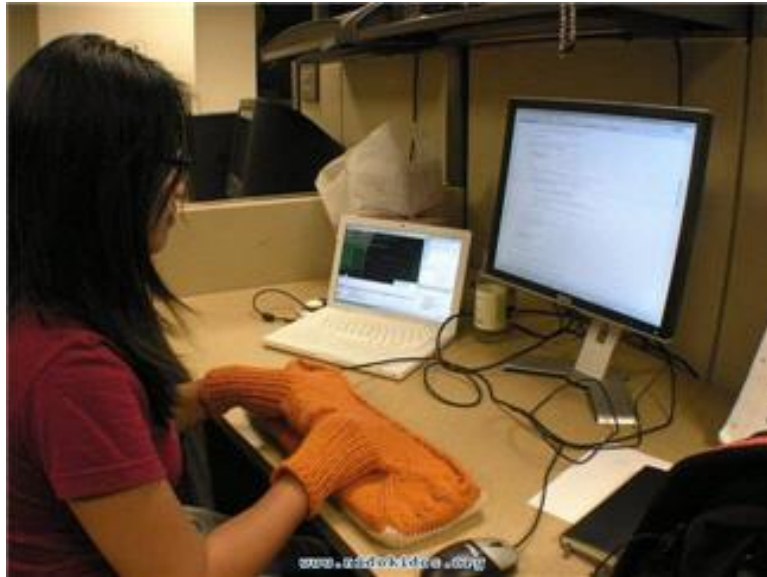
# The problem/system under study

# The threat model

- Who?
  - Alice and Eve
- What?
  - Alice is typing her password.
  - Eve is looking at Alice's fingers.
- How?
  - Eve is behind/beside Alice.
  - Eve installed a hidden camera.
  - Eve's malware in Alice's PC.
  - ...
- So, we need a solution!



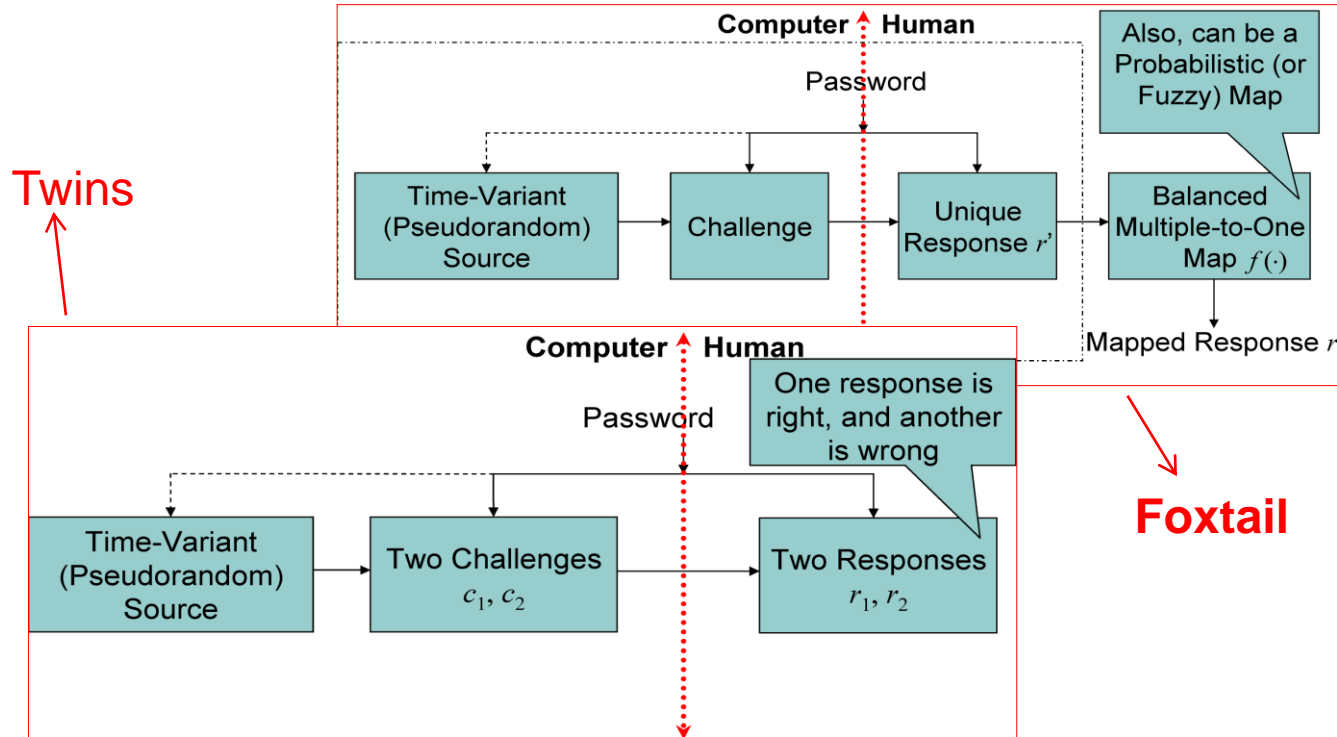
# Maybe something like these?



<http://www.isgafrika.org/blog/?p=138>

# Two basic structures

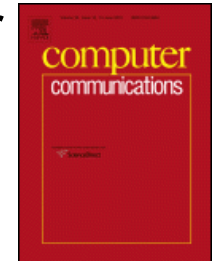
- Proposed by Li & Shum in 2002



# The virtual password system (VPS)

## - Who and where?

- Proposed by Lei et al. at ICC 2008 (two papers)
- A journal edition was published later in *Computer Communications* (also in 2008)



## - What and how?

### - "Virtual password"



- A static password  $X = x_1 x_2 \dots x_n$  in  $\mathbb{Z}^n$ , where  $\mathbb{Z} = \{0, \dots, \mathbb{Z}-1\}$
- A virtual function  $\mathfrak{B}: \mathbb{Z}^{2n} \rightarrow \mathbb{Z}^n$

### - Login = A challenge-response (Foxtail) protocol

- The challenge: A "random salt"  $Y = y_1 y_2 \dots y_n$  in  $\mathbb{Z}^n$
- The response: A "dynamical password"  $K = k_1 k_2 \dots k_n = \mathfrak{B}(X, Y)$



# VPS based on randomized linear generation function

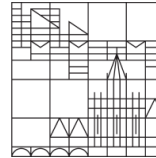
- The virtual function
  - A randomized linear function modulus an integer  $Z$
- The static password
  - The password string  $X=x_1x_2\dots x_n$  in  $\mathbb{Z}^n$
  - A secret integer  $a$  in  $\mathbb{Z}$
- The login process
  - Server  $\Rightarrow$  User:  $Y=y_1y_2\dots y_n$
  - User: A random integer  $c$  in  $\mathbb{Z}$ 
    - $k_1=\mathfrak{B}_1(X,a, Y,c)=(ax_1+y_1+x_2+c) \bmod Z$
    - $k_i=\mathfrak{B}_i(X,a,k_{i-1}, Y,c)=(ak_{i-1}+y_i+x_i+c+x_{i+1}) \bmod Z$ , for  $i=2,\dots,n-1$
    - $k_n=\mathfrak{B}_n(X,a,k_{n-1}, Y,c)=(ak_{n-1}+y_n+x_n+c+x_1) \bmod Z$
  - User  $\Rightarrow$  Server:  $K=k_1\dots k_n$



# A reported brute-force attack



- Who and where?
  - Coskun & Herley at **ISC 2008**  
INFORMATION SECURITY CONFERENCE
- How?
  - For  $n$  login sessions, exhaustive search  $X, \alpha, Y, c$  and intersect the result to get the password
    - $O(nZ^4) \Rightarrow O(nZ^3) \Rightarrow O(nZ^2) \Rightarrow O(nZ) \Rightarrow O(n)$
- What's the problem?
  - $c$  is session-varying so simple intersection does not work.
  - It can be fixed, but the attack will become more complicated.



# Our proposed attack

# Getting an equivalent password (1)

- The idea
  - We can turn to break an equivalent password  $X^*, a$
- Observed login sessions
  - Login session 1:  $c, Y=y_1 \dots y_n \Rightarrow K=k_1 \dots k_n$
  - Login session 2:  $c', Y'=y'_1 \dots y'_n \Rightarrow K'=k'_1 \dots k'_n$
- Breaking the secret integer  $a$ 
  - $a(k'_{i-1} - k_{i-1}) \equiv (k'_i - k_i) - (y'_i - y_i) - (k'_1 - k_1) + (y'_1 - y_1) \pmod{Z}$
  - If  $\gcd(k'_{i-1} - k_{i-1}, Z) = 1$ , then
 
$$a = (k'_{i-1} - k_{i-1})^{-1} ((k'_i - k_i) - (y'_i - y_i) - (k'_1 - k_1) + (y'_1 - y_1)) \pmod{Z}$$
  - Probability =  $\varphi(Z)/Z = \prod_{p|Z} (1 - p^{-1})$
  - Complexity  $\leq O((\log Z)^2)$



# Getting an equivalent password (2)

- Deriving an equivalent password string  $X^* = x^*_1 \dots x^*_n$ 
  - $x^*_1 \equiv (ax_1 + x_2 + c) \equiv (k_1 - y_1) \pmod{Z}$
  - $x^*_i \equiv (x_i + x_{i+1} + c) \equiv (k_i - ak_{i-1} - y_i) \pmod{Z}$ , for  $i=2, \dots, n-1$
  - $x^*_n \equiv (x_n + x_1 + c) \equiv (k_n - ak_{n-1} - y_n) \pmod{Z}$
  
- To impersonate the legitimate user
  - Pick a random integer  $c^*$  in  $Z$
  - Send a response  $K^* = k^*_1 \dots k^*_n$  to the server
    - $k^*_1 = (x^*_1 + y_1 + c^*) \pmod{Z}$
    - $k^*_i = (ak^*_{i-1} + y_i + x^*_i + c^*) \pmod{Z}$ , for  $i=2, \dots, n$

# Getting an equivalent password (2)

- Deriving an equivalent password string  $X^* = x^*_1 \dots x^*_n$ 
  - $x^*_1 \equiv (\alpha x_1 + x_2 + c) \equiv (k_1 - y_1) \pmod{Z}$
  - $x^*_i \equiv (x_i + x_{i+1} + c) \equiv (k_i - \alpha k_{i-1} - y_i) \pmod{Z}$ , for  $i=2, \dots, n-1$
  - $x^*_n \equiv (x_n + x_1 + c) \equiv (k_n - \alpha k_{n-1} - y_n) \pmod{Z}$
  
- To impersonate the legitimate user
  - Pick a random integer  $c^*$  in  $Z$
  - Send a response  $K^* = k^*_1 \dots k^*_n$  to the server
    - $k^*_1 = (x^*_1 + y_1 + c^*) \pmod{Z}$
    - $k^*_i = (\alpha k^*_{i-1} + y_i + x^*_i + c^*) \pmod{Z}$ , for  $i=2, \dots, n$

# Breaking the original password?

- It is possible to break  $X$  in most cases!

- How?

- $\mathbf{X}^* \equiv \mathbf{A}\mathbf{X} + c \pmod{Z}$ , where  $\mathbf{A} = \begin{bmatrix} a & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 1 \\ 1 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}$

- $|\mathbf{A}| = a-1$  (when  $n$  is odd) or  $a+1$  (when  $n$  is even)
- If  $g = \gcd(|\mathbf{A}|, Z) = 1$ :  $\mathbf{X} = (\mathbf{A}^{-1}(\mathbf{X}^* - c)) \pmod{Z}$
- If  $g = \gcd(|\mathbf{A}|, Z) > 1$ :  $\mathbf{X} \equiv (|\mathbf{A}|/g)^{-1} \text{adj}(\mathbf{A})(\mathbf{X}^* - c) \pmod{Z/g}$ 
  - If  $g^n < Z^{n-1}$ : The password space can be narrowed down to reveal  $X$  with  $m > 1$  observed login sessions.

# A simple example

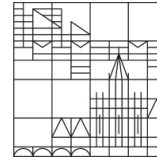


- Parameters:  $Z=10, n=4$
- Password:  $\alpha=7, X=1234$
- Two observed login sessions
  - Login session 1:  $Y=1674 \Rightarrow K=3526$
  - Login session 2:  $Y'=6837 \Rightarrow K'=4094$
- Getting the equivalent password  $(\alpha, X^* = x^*_1 x^*_2 x^*_3 x^*_4)$ 
  - $\alpha = \gcd(4-3, 10)^{-1}((0-5)-(8-6)-(4-3)+(6-1)) \bmod 10 = 7 \checkmark$
  - $x^*_1 = (3-1) \bmod 10 = 2$
  - $x^*_2 = (5-7*3-6) \bmod 10 = 8$
  - $x^*_3 = (2-7*5-7) \bmod 10 = 0$
  - $x^*_4 = (6-7*2-4) \bmod 10 = 8$

- Success rate of 1000 random attacks ( $n=4$ )

	$m=2$	$m=3$	$m=4$	$m=5$	$m \geq 6$
$Z = 10$	0.791	0.984	0.998	1	1
$Z = 26$	0.838	0.988	0.999	1	1
$Z = 36$	0.699	0.960	0.995	0.998	1
$Z = 52$	0.839	0.985	1	1	1
$Z = 62$	0.865	0.981	0.998	1	1
$Z = 95$	0.985	1	1	1	1

- Try our MATLAB code yourself
  - <http://www.hooklee.com/Papers/Data/VPS.zip>
  - Run `RandomAttack` to simulate an attack
  - Run `SuccessRates` to estimate the success rate of a number of random attacks



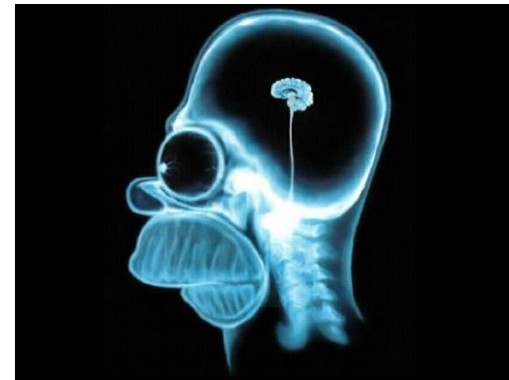
# Take-home message (with related work)

# This is an unsolved open problem for nearly 20 years!

## - Security vs. Usability



## - Rich and smart attackers vs. Poor and “stupid” users



# Related work: Is it indeed that difficult?

- T. Matsumoto and H. Imai, "Human identification through insecure channel," EUROCRYPT'91

Question	Answer																																
<p>Hello!</p> <p>Please fill the boxes using characters from {1,2,3,4,5,6,7,8,9,0}.</p> <p>q = <table border="1"><tr><td>2</td><td>8</td><td>5</td><td>1</td><td>7</td><td>3</td><td>6</td><td>4</td></tr></table></p> <p>a = <table border="1"><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table></p>	2	8	5	1	7	3	6	4									<p>Hello!</p> <p>Please fill the boxes using characters from {1,2,3,4,5,6,7,8,9,0}.</p> <p>q = <table border="1"><tr><td>2̄</td><td>8</td><td>5</td><td>1̄</td><td>7</td><td>3</td><td>6̄</td><td>4̄</td></tr></table></p> <p>a = <table border="1"><tr><td></td><td>3</td><td>1</td><td>2</td><td>1</td><td>2</td><td>4</td><td></td></tr></table></p> <p><math>\Lambda = \{1, 2, 4, 6\}, \Delta = \{1, 2, 3, 4\}</math> <math>W = 3124</math></p>	2̄	8	5	1̄	7	3	6̄	4̄		3	1	2	1	2	4	
2	8	5	1	7	3	6	4																										
2̄	8	5	1̄	7	3	6̄	4̄																										
	3	1	2	1	2	4																											

**Broken by Wang et al. (EUROCRYPT'95)**

# Related work: Is it indeed that difficult?

- T. Matsumoto, "Human-computer cryptography: An attempt," CCS'96

Please answer the number of winners.

A B C D E F G H I  
A B C D E F G H I  
A B C D E F G H I

0 winners  
1 winner  
2 winners

**Insecure against multiple observations**

# Related work: Is it indeed that difficult?

- N. J. Hopper and M. Blum, “Secure human identification protocols,” ASIACRYPT’2001
  - Two protocols based on mathematical hard problems
  - $\Rightarrow$  Security may be ensured (if the underlying problem is indeed hard)
- Require the user to make deliberate errors and/or perform nontrivial computations
- $\Rightarrow$  Usability is problematic (166 seconds for login for an implementation reported in HB paper)

# Related work: Is it indeed that difficult?

- D. Weinshall, "Cognitive authentication schemes safe against spyware," Oakland'2006



**Broken by Golle & Wagner  
(Oakland'2007)**

# Related work: Is it indeed that difficult?

- Xiaole Bai et al., "PAS: Predicate-based Authentication Services Against Powerful Passive Adversaries," ACSAC'2008

(1,3) DFGHKR	(1,2) ABDFGL	(1,3) ABFGJKL	(1,4) DGHLMN	(1,5) CDEFKM	
TUVWXYZ	MORSUWY	NSUWXZ	PRUVWXZ	OPSTUXZ	
(2,3) DEFHJK	(2,2) CHKLNO	(2,3) CEHLNO	(2,4) DEFGJK	(2,5) ABCDEF	
OPSTUVW	PQRVXYZ	RSUWXYZ	OQSTVYZ	GKLMORX	
(3,1) AFGHJK	(3,2) AEFHKQ	(3,3) BCEFHJL	(3,4) AEGE		
MOQRSTV	RSUWXYZ	OPQUWZ	MOQTU		
(4,3) ABEFGJK	(4,2) BCDEFH	(4,3) AGHJKM	(4,4) ABCD		
NPSTXZ	MQSTUXY	NPQTUWY	LMNOF		
(5,1) ACEGKM	(5,2) CDEFGH	(5,3) BCHKMN	(5,4) CDEF		
NORTWXY	JMOQSTU	RTVWXYZ	MQRST		
(1,1) CEHKLM	(1,2) CEKLNO	(1,3) ABEGKL	(1,4) ACFL		
NPQRUVW	PQRSVYZ	OQSTVWY	PQRSU		
(2,1) BCEFMO	(2,2) ACDEFJN	(2,3) ACEHJM	(2,4) ACDC		
PQSTVWY	OPQSTX	NPQTUYZ	KLNQS		
(3,1) BCDFHJ	(3,2) ADEFGH	(3,3) ABEJLNQ	(3,4) ADEG		
MNQRSVY	LMPQRUY	RSVWXY	NOPQR		
(4,1) BDEKOP	(4,2) ACEFK	(4,3) BEGKO	(4,4) BDEIKL	(4,5) BGHJF	
QSTUVXZ	NPRSTVW	TUVWZ	RSUVW	QSVVX	
(5,1) BCDEFLN	(5,2) CDJKNO	(5,3) BHIQ	(5,4) CEKLN	(5,5) AFHJ	
PQRUVX	PQSUXYZ	TRSTVYZ	QRTUVW	NPRVWXZ	

	2: No No	2: No Yes	2: Yes No	2: Yes Yes
1: No No				
1: No Yes				
1: Yes No				
1: Yes Yes				

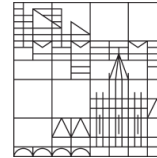
**Broken by us  
(ACSAC'2009)**

# Related work: Is it indeed that difficult?

- Hirokazu Sasamoto, Nicolas Christin and Eiji Hayashi, “Undercover: Authentication Usable in Front of Prying Eyes,” CHI’2008



**To be broken by us  
(CHI'2011?)**



# Thanks for your attention!

## It's time for questions and answers 😊

